Breaking the mass-sheet degeneracy with gravitational wave interference in lensed events

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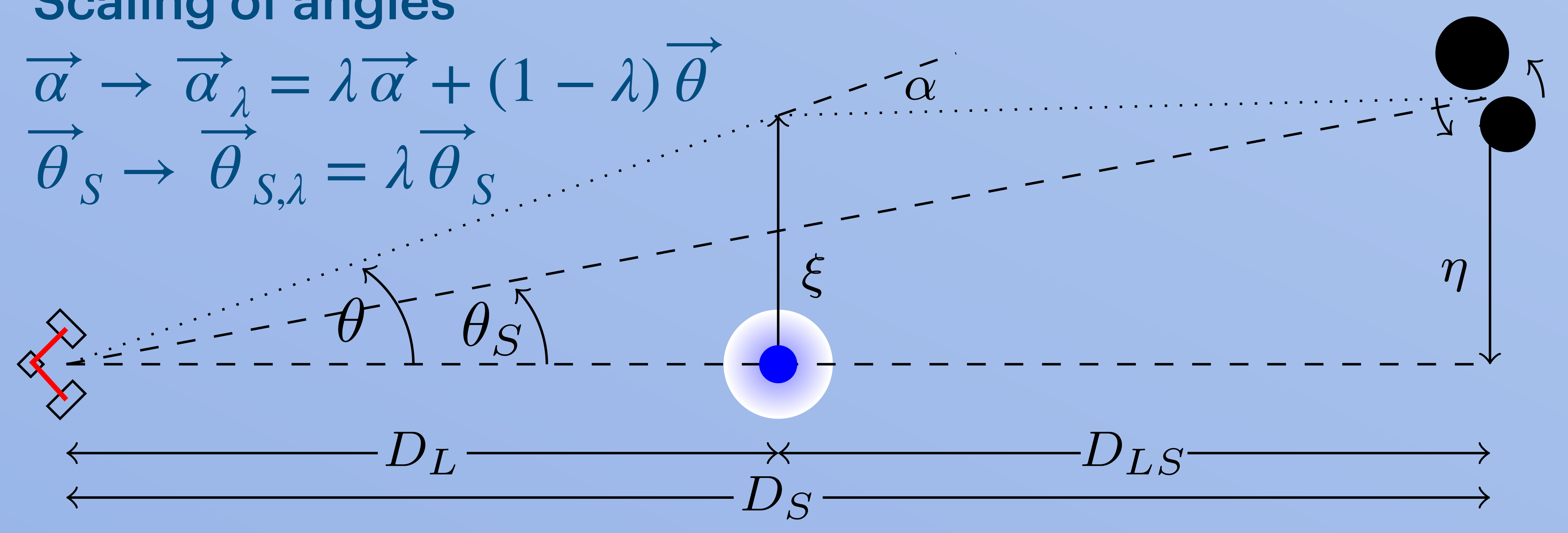
1. Background

The MSD is a well-known problem in lensing of light.
It consists on [1]

- Scaling of lens mass
  (κ - surface mass density)
  $\kappa \to \kappa_\lambda = \lambda\kappa + (1-\lambda)$
- Scaling of angles
  $\vec{\alpha} \to \vec{\alpha}_\lambda = \lambda\vec{\alpha} + (1-\lambda)\vec{\theta}$
  $\vec{\theta}_S \to \vec{\theta}_{S,\lambda} = \lambda\vec{\theta}_S$

λ is the transformation/ degeneracy parameter
$\lambda = 1 \Rightarrow$ no transformation



2. Problem

- Observables are preserved!
- Biased estimations of lens parameters
- Biased estimation of cosmological parameter (e.g. H_0)

3. Solution

In lensing of light: multiple images; independent mass estimation of lens; multiple lenses.

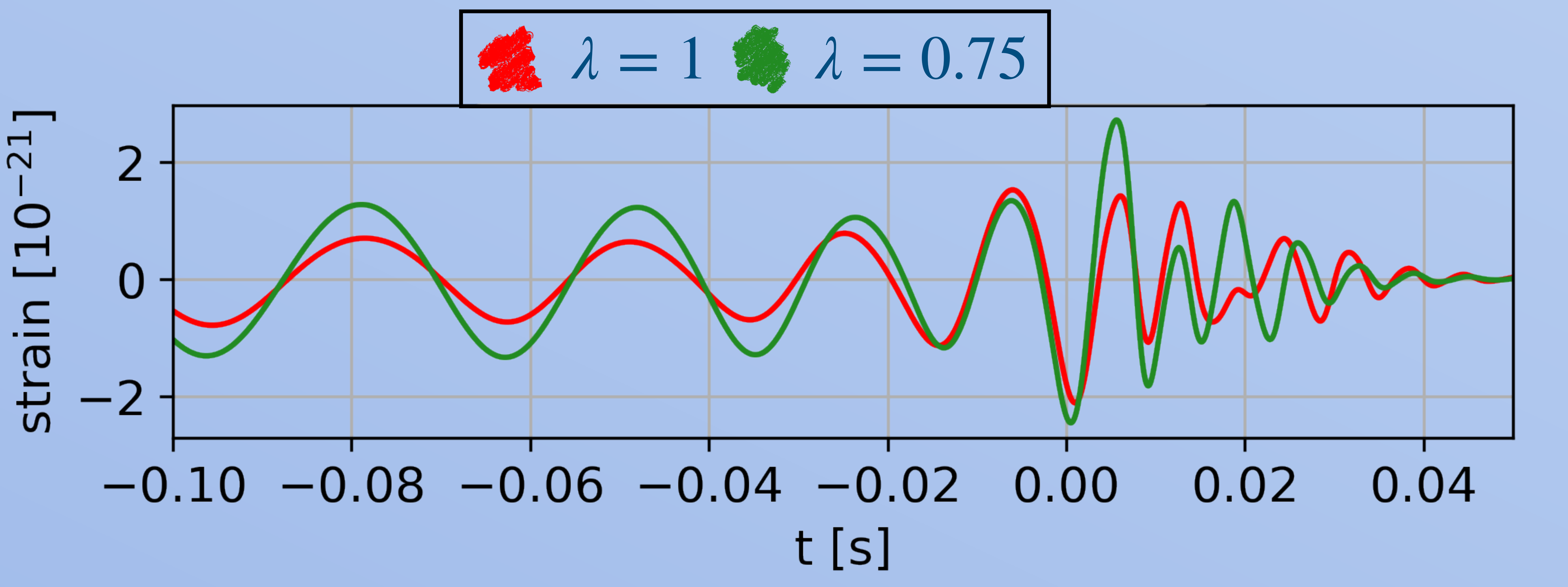
In GW lensing: 1 image and 1 lens can break MSD!

4. Gravitational Wave lensing

Lensed waveform in frequency domain $\tilde{h}_L(f)$ is:
$\tilde{h}_L(f) = \tilde{h}(f) \cdot F(f, \theta_s)$, with $\tilde{h}(f)$ the unlensed signal, f the GW frequency and $F(f, \theta_s)$ the amplification factor [2], which contains all info about lens.

5. MSD in GW lensing

According to the MSD transformation, also the amplification factor changes: $F \to F_\lambda$.
Signal with $M_S = 100\ M_\odot$ at $z_S = 0.1$ lensed by a point mass lens with $M_L = 500\ M_\odot$ & $y\ (\sim \theta_S) = 1$ at $z_L = 0.01$, gives



The shape of the lensed waveform depends on λ. Hence, the degeneracy is broken!
We studied, then, up to which level the MSD is broken.

6. Template matching (signal-to-noise ratio - SNR)

In a matched filtering analysis, the SNR is

$\rho \approx \frac{(h\,|\,h_T)}{\sqrt{(h_T\,|\,h_T)}}$, with h the signal,

h_T the template and $(a\,|\,b)$ the inner product.

$(a\,|\,b) = 4\,\mathrm{Re}\left[\int_0^\infty \frac{\tilde{a}(f)\cdot\tilde{b}^*(f)}{S_n(f)}df\right]$
with $S_n(f)$ - (single-sided) power spectral density (L1-O3-LIGO)

The confidence region is given by $\frac{\rho}{\rho_{opt}} = 1 - \left[\frac{1}{2}\frac{\Delta\chi^2}{\rho^2_{opt}}\right]$,

with $\Delta\chi^2 \approx 11.8$ at 3σ. Then, we compare the signal with the (supposedly) degenerate one, computing ρ/ρ_{opt}.

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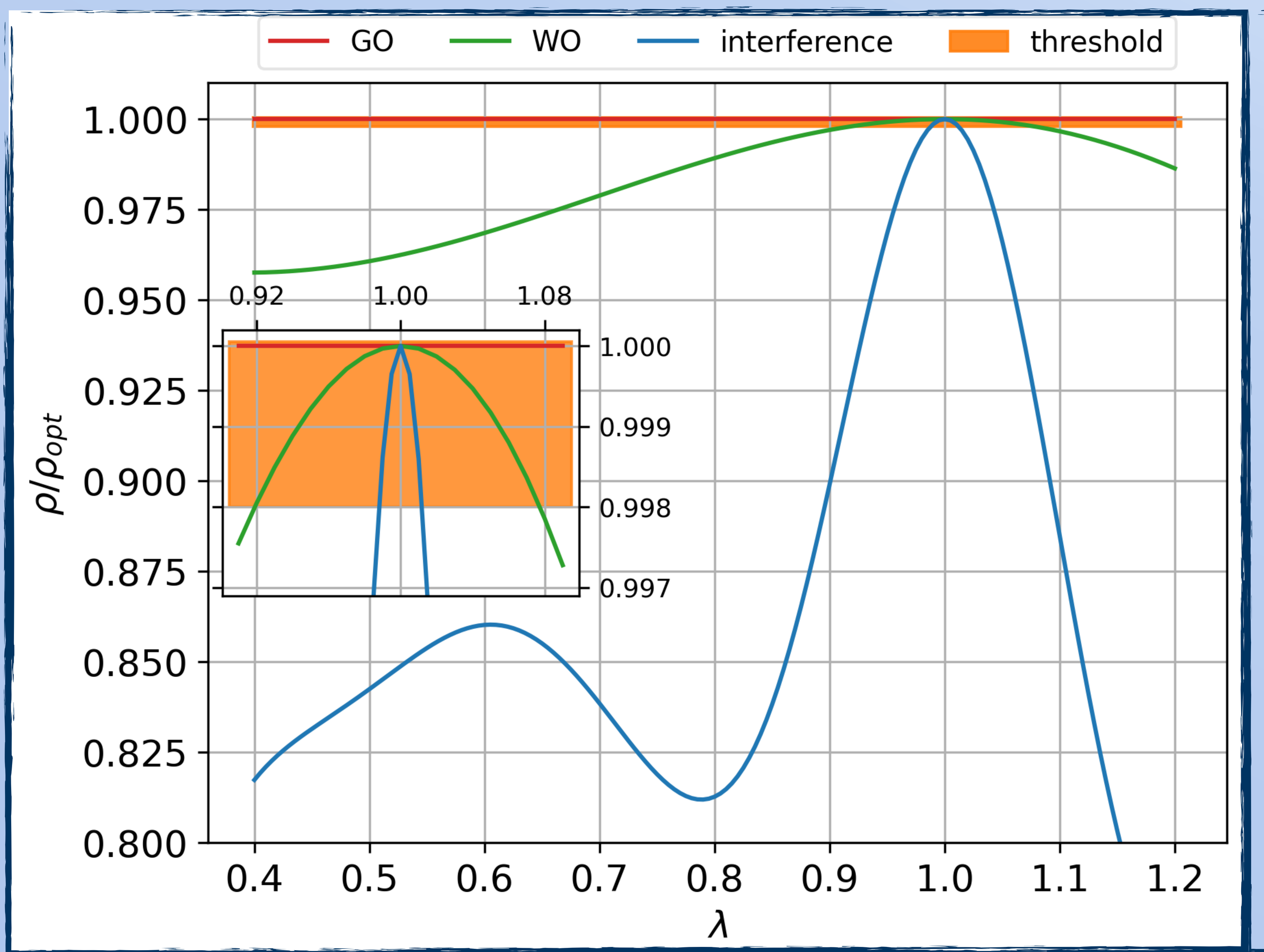

In the Figure, we show the behaviour of the match between the original signal and the degenerate one (ρ/ρ_{opt}) with λ.
When the curve leaves the confidence region, the MSD is broken, like in interference and wave-optics regimes (blue and green curves), but not in geometrical optics (red curve).

7. Results

The errors on the inferred parameters of the lens are:

- For a signal with $\rho = 11$, $\Delta y < 40\,\%$ and $\Delta M \approx 35\,\%$
- **For a signal with $\rho = 55$, $\Delta y \approx 5\,\%$ and $\Delta M \approx 6\,\%$**
- From dynamics calculations [4]: $\Delta M \approx 12 - 20\,\%$

8. Conclusions

- MSD ruins inference precision in lensing events
- In some cases of GW lensing (interference and wave-optics regimes), the MSD can be broken
- How well it is broken depends on the strength of the signal and sensitivity of detectors.

Bibliography

[1] E. E. Falco, M. V. Gorenstein, and I. I. Shapiro, ApJ289, L1 (1985)
[2] T. T. Nakamura and S. Deguchi, "Progress of Theoretical Physics Supplement" 133, 137 (1999).
[3] M. Maggiore, "Gravitational Waves: Volume 1" (OUP Oxford, 2008)
[4] P. Schneider and D. Sluse, Astron. Astrophys. 559, A37(2013)