High accuracy on H_0 measurements from gravitational wave lensing events

P. Cremonese (paolo.cremonese@usz.edu.pl) & V. Salzano
Institute of Physics University of Szczecin, Wielkopolska 15, 70-451 Szczecin, Poland

What?

We investigate the possibility to achieve high precision on H_0 measurements by using gravitational waves (GWs) in an alternative way w.r.t. current methods, like [1] and [2] which give respectively $H_0 = 70^{+12}_{-8}$ and 68^{+14}_{-7}.

In case of a multi-messenger detection and a contemporary gravitational lensing event, we might measure the arrival time difference between the lensing time delay of the GW and of the electromagnetic (EM) counterpart and from that infer H_0.

Geometrical optics - EM signal

The EM signal can be described by standard geometrical optics as [3]:

$$t_{EM}(\vec{\theta}, \vec{\beta}) = \frac{1+z_l}{c} \frac{D_l D_s}{D_{ls}} \left[\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta}) \right]. \tag{1}$$

Wave optics - GW

If the mass of the lens $M_{lens} < 10^5 M_\odot (f/Hz)^{-1}$, where f is GW frequency, the time delay must be defined using wave optics [3] as:

$$T_{GW}(w, y) \equiv -\frac{i}{w} \ln \left(\frac{F(w,y)}{|F(w,y)|} \right), \tag{2}$$

with: F, the amplification factor $F(w,y) = \frac{w}{2\pi i} \int d^2x \exp[iwT_{EM}(x,y)]$; T_{EM} and T_{GW}, dimensionless times, by multiplying dimensional times to the factor $\frac{cD_{ls}}{D_l D_s} \theta_*^{-2} (1+z_l)^{-1}$; $x = \theta/\theta_*$, image position; $y = \beta/\theta_*$, source real position; $w = \frac{D_l D_s}{c D_{ls}} \theta_*^2 (1+z_l) 2\pi f$, dimensionless frequency; and θ_*, a characteristic radius of the lens depending on the mass model.

Arrival time difference

Thus, the arrival time difference is:

$$\Delta T_{\rm EM-GW}(y, w) = T_{\rm EM}(x, y) - T_{\rm GW}(y, w). \tag{3}$$

$\Delta T_{\rm EM-GW}$ has no x-dependence because we calculate x from the lens equation [4], providing y.

How?

Eqs. (1) and (2) depend on the cosmological model through the angular diameter distances and the lens potential ψ. The Hubble parameter is

$$H^2(z) = H_0^2 \cdot [\Omega_\gamma (1+z)^4 + \Omega_m (1+z)^3 + \Omega_{DE} (1+z)^{3(1+w)}]. \tag{4}$$

We consider ΛCDM ($w = -1$) and quiessence ($w = const. \neq -1$).

Methodology

We proceed as follows:

1. we calculate $\Delta T_{\rm EM-GW}$ for a large set of input $\{\Omega_m, H_0, w\}$;
2. we assume an independent prior on Ω_m from *Planck*, $\Omega_m = 0.3061 \pm 0.0052$;
3. we infer the uncertainty on H_0 by crossing the above prior with the arrival time uncertainty.

A crucial ingredient is the uncertainty on the GW time delay. We consider the Pulsar Timing Array (PTA), for which the error is $\sigma_{\Delta T} = (2\pi f \rho^2)^{-1}$, with the S/N ratio ρ^2 being [5]:

$$\rho^2 = \hat{\rho}^2 \cdot (1+z)^4 \left(\frac{f_{\rm orb}}{f_{\rm obs}} \right)^{-2/3}, \tag{5}$$

where $f_{\rm orb}$ is the orbital frequency of the Super Massive Binary Black Hole (SMBBH) emitting the radiation and $f_{\rm obs}$ is the lowest frequency detectable by the PTAs, and with

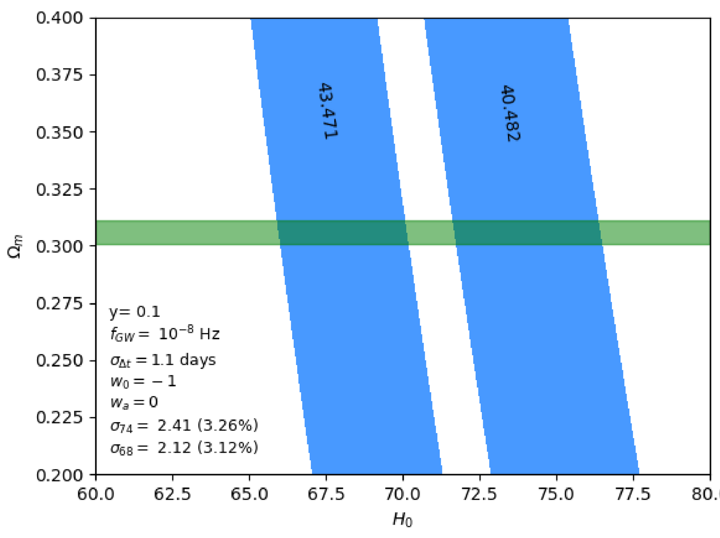
$$\hat{\rho}^2 = 4.26 \cdot 10^{-2} N_{\rm p} (N_{\rm p} - 1) \left(\frac{\mathcal{M}}{10^8 M_\odot} \right)^{10/3} \times \\ \times \left(\frac{T_{\rm obs}}{10 \text{ yr}} \right)^{5/3} \left(\frac{100 \text{ Mpc}}{d_{\rm L}} \right)^2 \left(\frac{100 \text{ ns}}{\sigma_{\rm rms}} \right) \left(\frac{0.05 \text{ yr}}{\Delta t} \right). \tag{6}$$

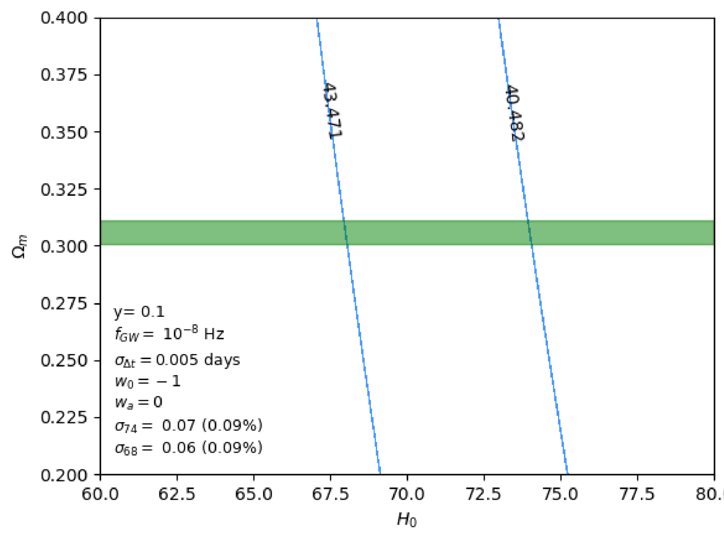
We study two scenarios: $i)$ a state-of-the-art sample made of 65 pulsars [6]; $ii)$ an "optimistic" future sample of 1000 pulsars detected by SKA [7]. Moreover, we also vary: the redshift of the source: $z_l = 0.5, 1$; the real position of the source: $y = 0, 0.1, 1$; and the mass model for the lens, using a Singular Isothermal Sphere (SIS) and a Navarro-Frank-White (NFW). For SIS we consider a typical stellar dispersion velocity $\sigma_* = 220$ km/s; for NFW we assume a realistic model as observed in [8].

z_l	$\mathcal{M}$ $(10^8 M_\odot)$	T_{obs} (yr)	σ_{rms} (ns)	Δt (yr)	N_p	$\sigma_{\Delta T}$ (days)
0.5 (1)	5	10	100	0.05	1000	0.005 (0.008)
					65	1.099 (1.883)

Results

- ΛCDM: with ~ 65 pulsar, we could match current precision on H_0 from [9]. SKA will improve it by 2 order of magnitudes;
- quiessence: we need to combine multiple measurements to decrease the error, at least $n \sim 10$; but the probability of detection is low [3].





z_l	0.5				1			
$\sigma_{\Delta T}$ (days)	1.1		0.005		1.88		0.008	
$y \downarrow \mid H_0 \rightarrow$	68	74	68	74	68	74	68	74
	NFW - $w = -1$							
0.1	**2.12**	**2.41**	0.06	0.07	**3.4**	**3.85**	0.06	0.07
1	5.5	5.9	**0.08**	**0.08**	8.9	7.65	**0.09**	**0.1**
	NFW - w free							
0	14.80	16.20	12.70	14.20	15.10	16.90	12.70	13.90
1	17.50	18.90	13.20	14.30	nd	nd	12.70	13.90
	SIS - $\sigma_* = 220$ (km/s) - $w = -1$							
0.1	**4.05**	**4.5**	0.07	0.08	**6.6**	**6.45**	0.08	0.09
1	10	10	**0.13**	**0.13**	10	10	**0.21**	**0.2**
	SIS - $\sigma_* = 220$ km/s - w free							
0	16.40	18.20	13.10	14.30	17.00	19.40	12.70	13.90
1	nd	nd	13.30	14.40	nd	nd	12.90	14.10

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