Wave-optics in Gravitational Waves lensed events

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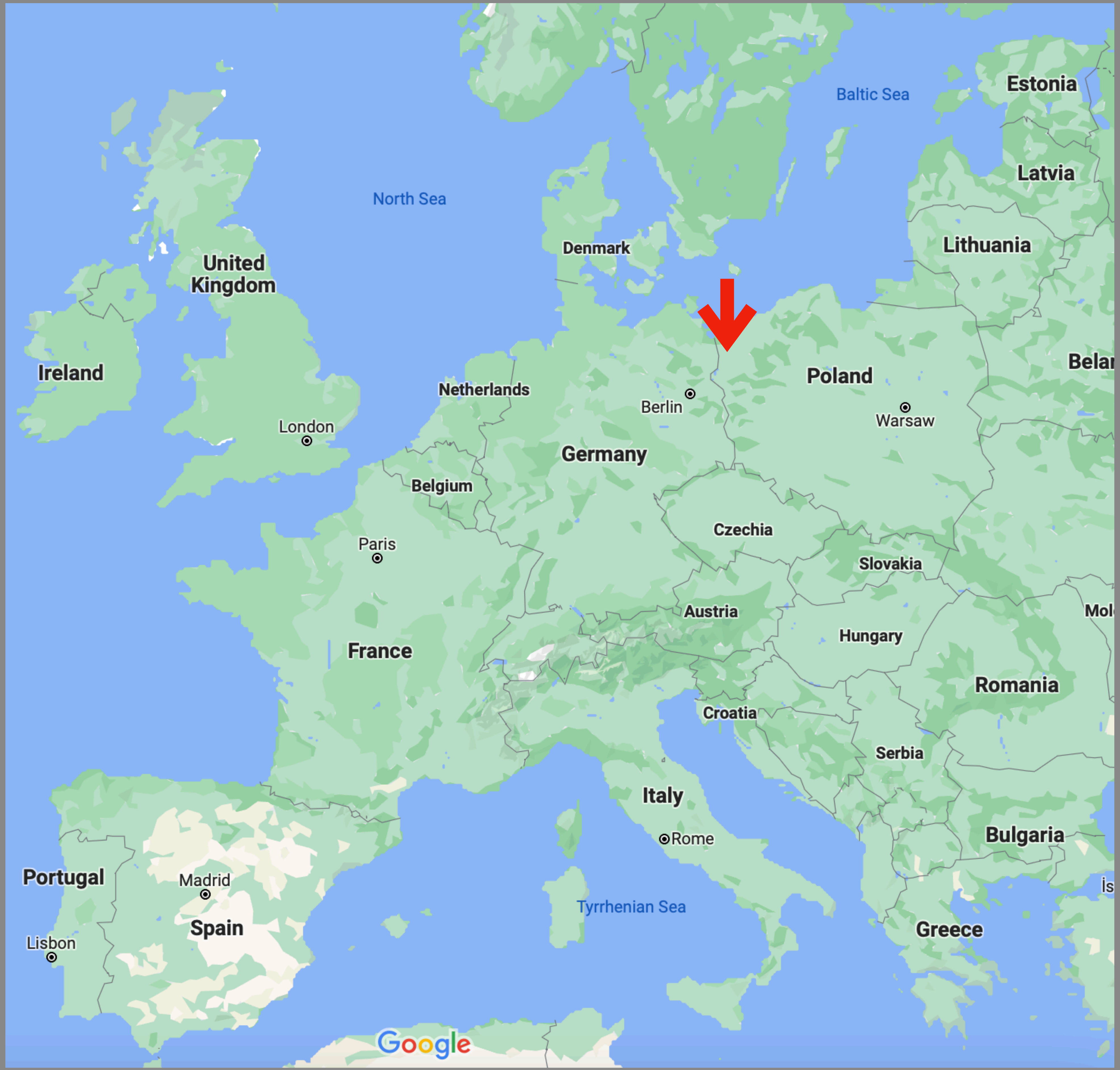
Based on
arXiv:1911.11786
arXiv:2104.07055
arXiv:2111.01163

05/10/2021

Szczecin cosmology group

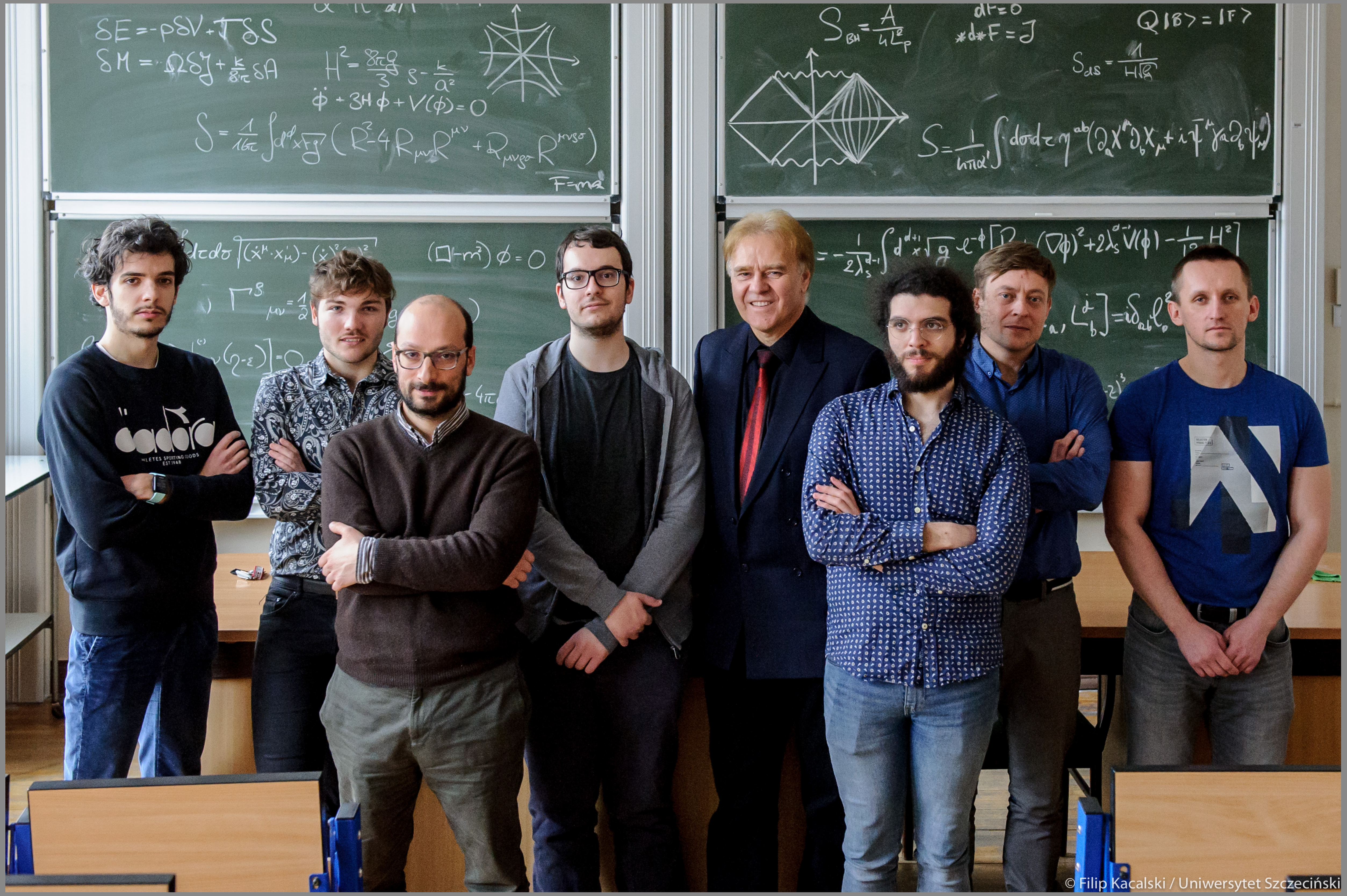
Szczecin cosmology group

Where is Szczecin?



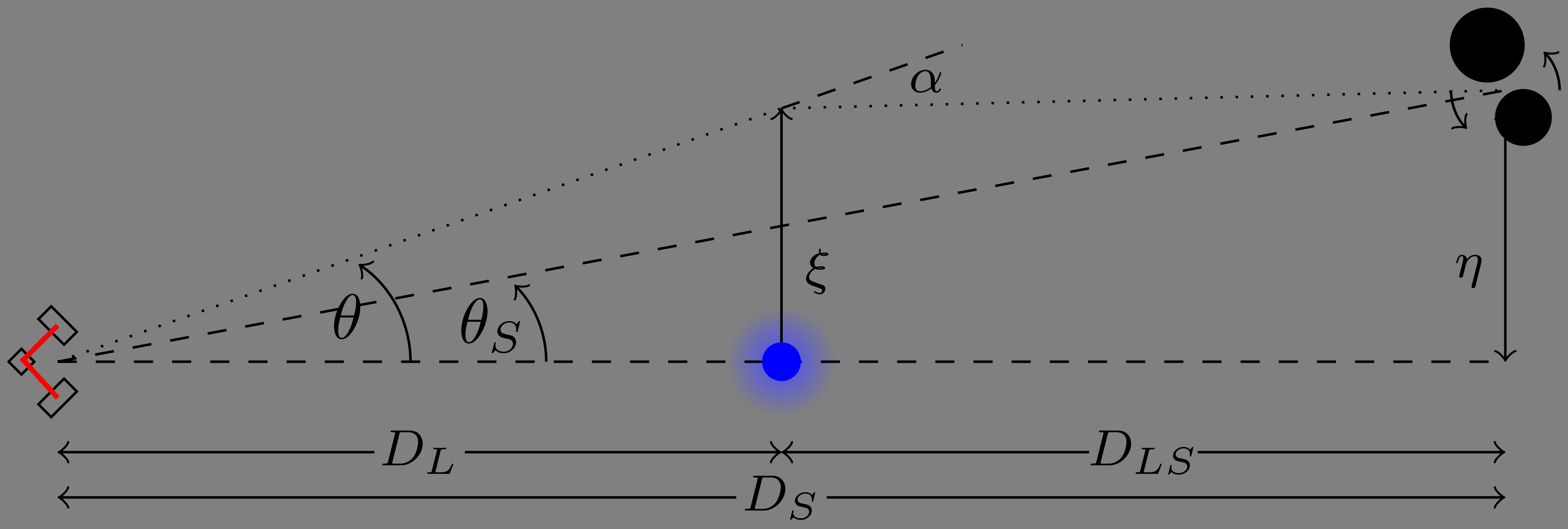
Cosmology group

cosmo.usz.edu.pl

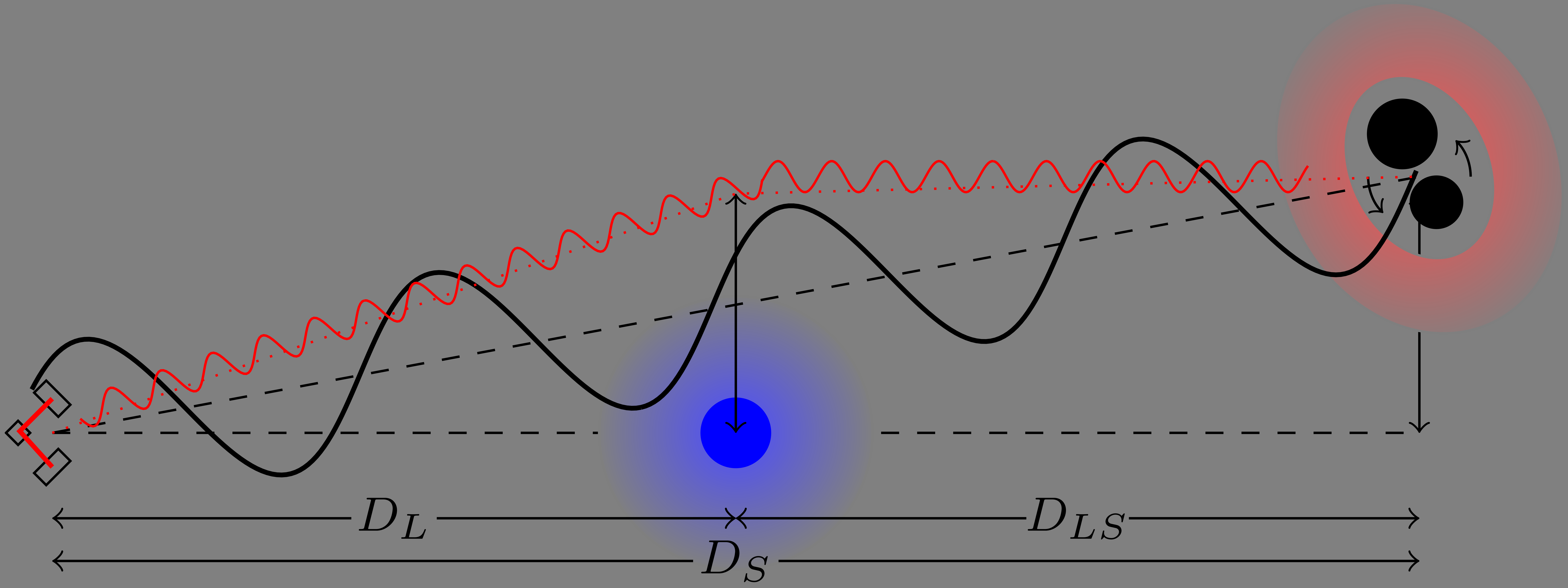


Gravitational Wave lensing

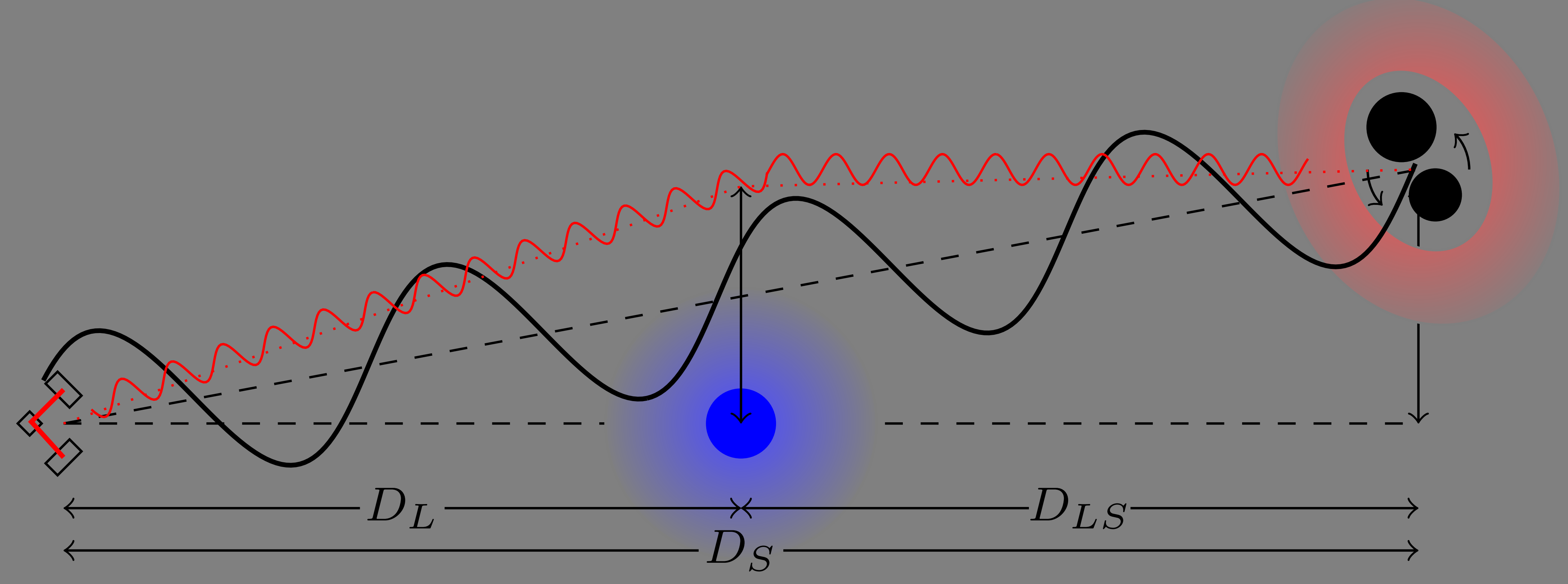
Gravitational Wave lensing



Gravitational Wave lensing



Geometrical-optics vs wave-optics



Geometrical-optics approximation breaks when

$$M_{3D,L} \leq 10^5 M_\odot \left[\frac{(1+z_L)f}{\text{Hz}}\right]^{-1}$$

$$f \cdot \Delta t \leq 1$$

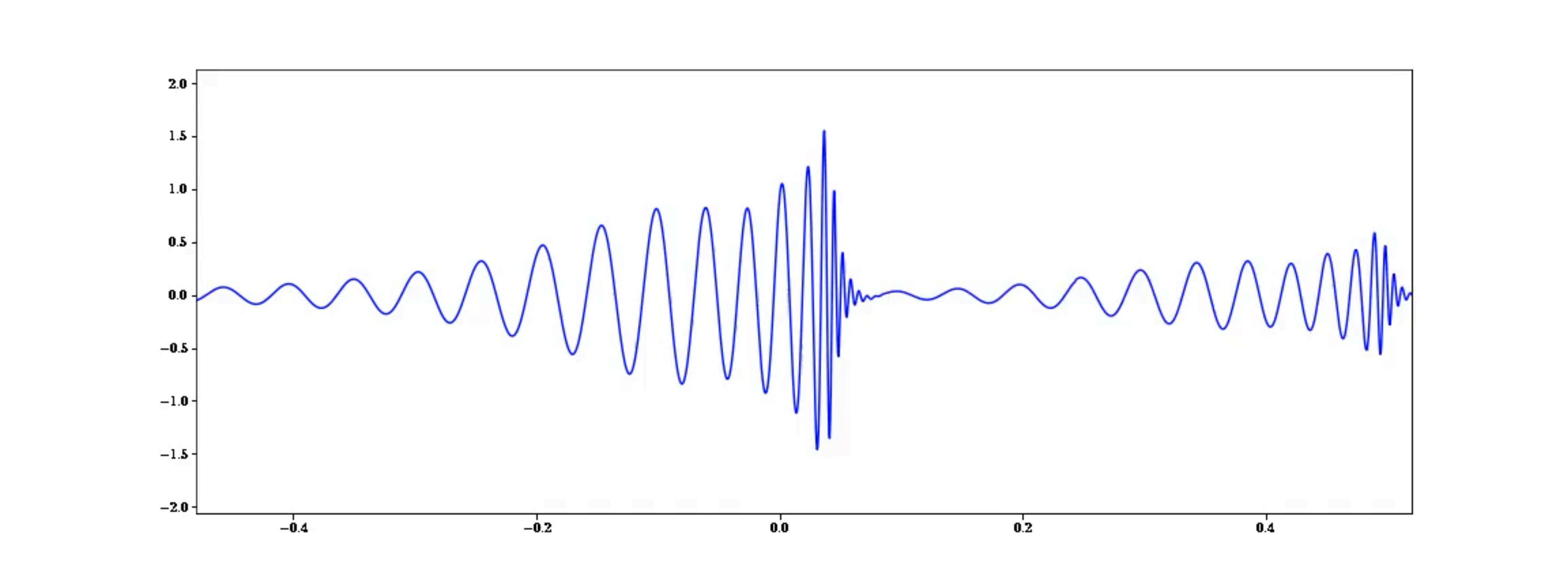
R.Takahashi,Astrophys.J.835,103(2017),arXiv:1606.00458 [astro-ph.CO]

$$\text{LHS} = 10^4\ M_\odot$$

$$f \approx 10^2\ \text{Hz}$$

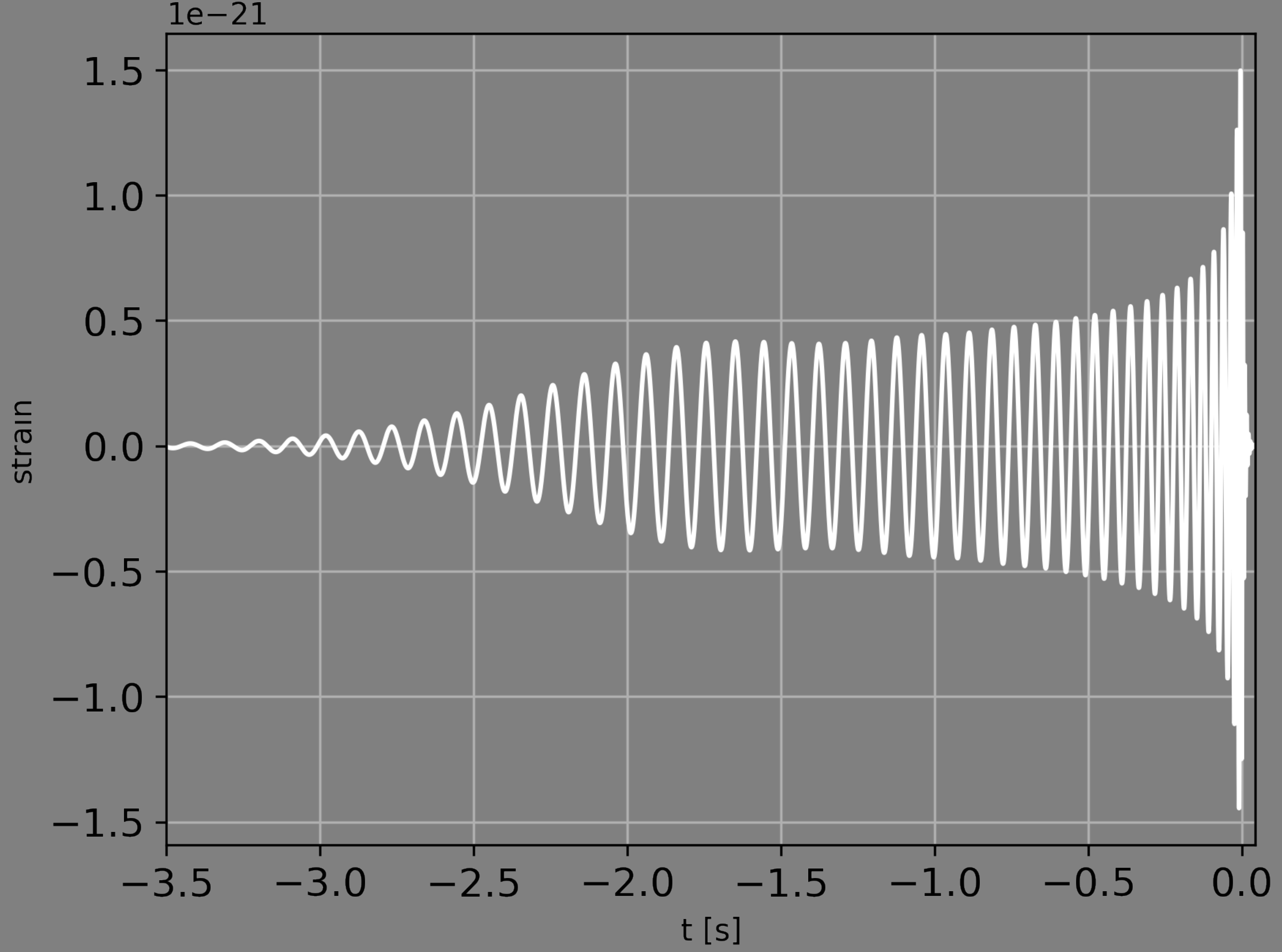
$$\text{RHS} = 10^3\ M_\odot$$

GO stands!



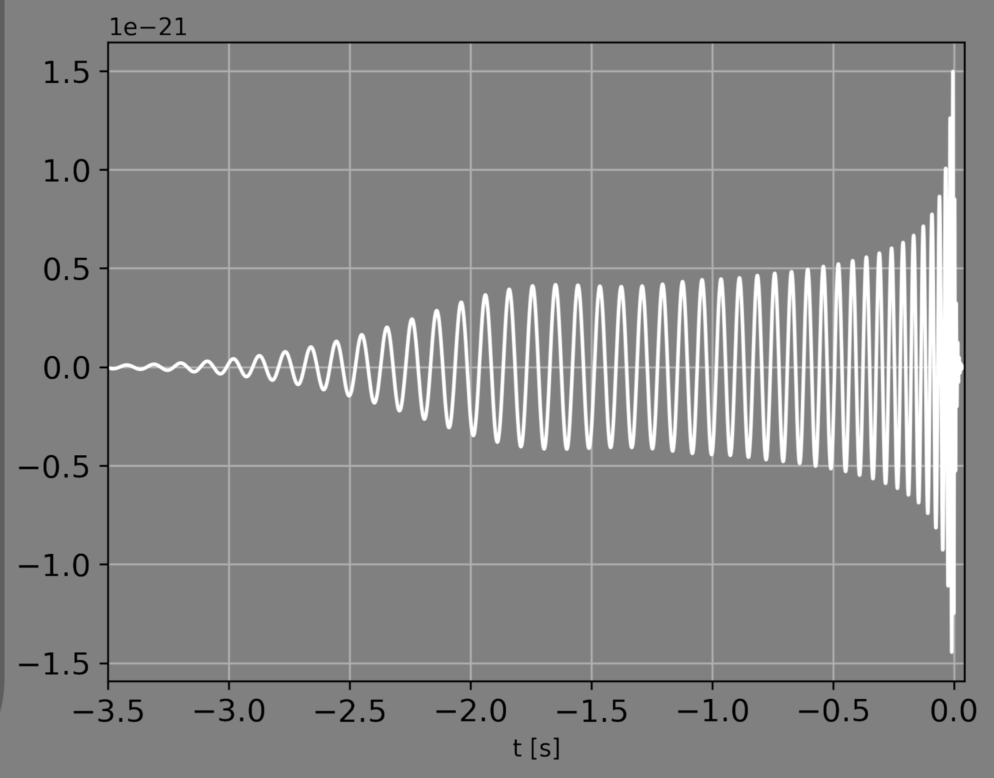
GL of GW

$$h(t)$$

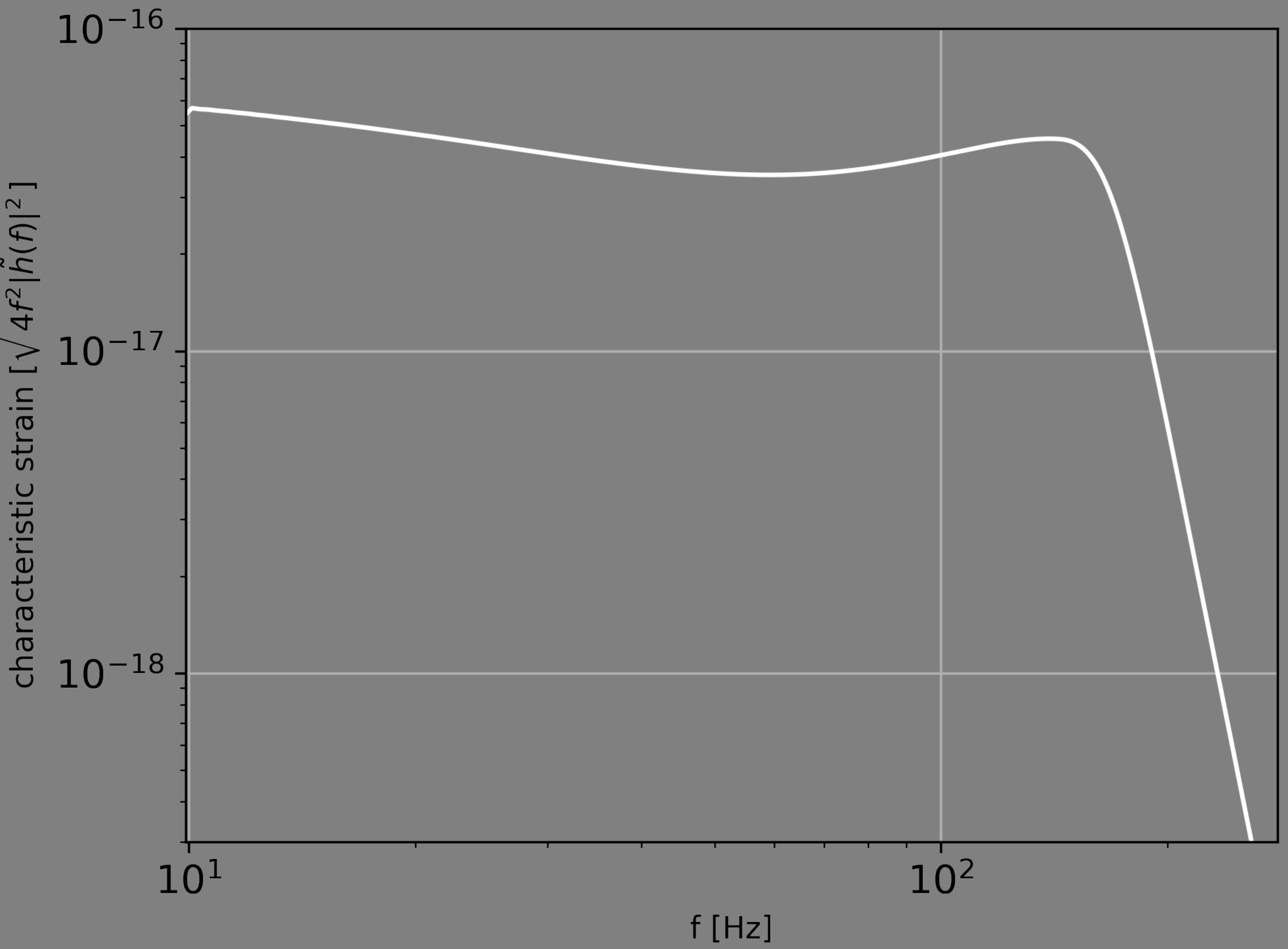


GL of GW

$$\int_{-\infty}^{\infty} h(t) \cdot e^{-i2\pi ft} dt = \tilde{h}(f)$$

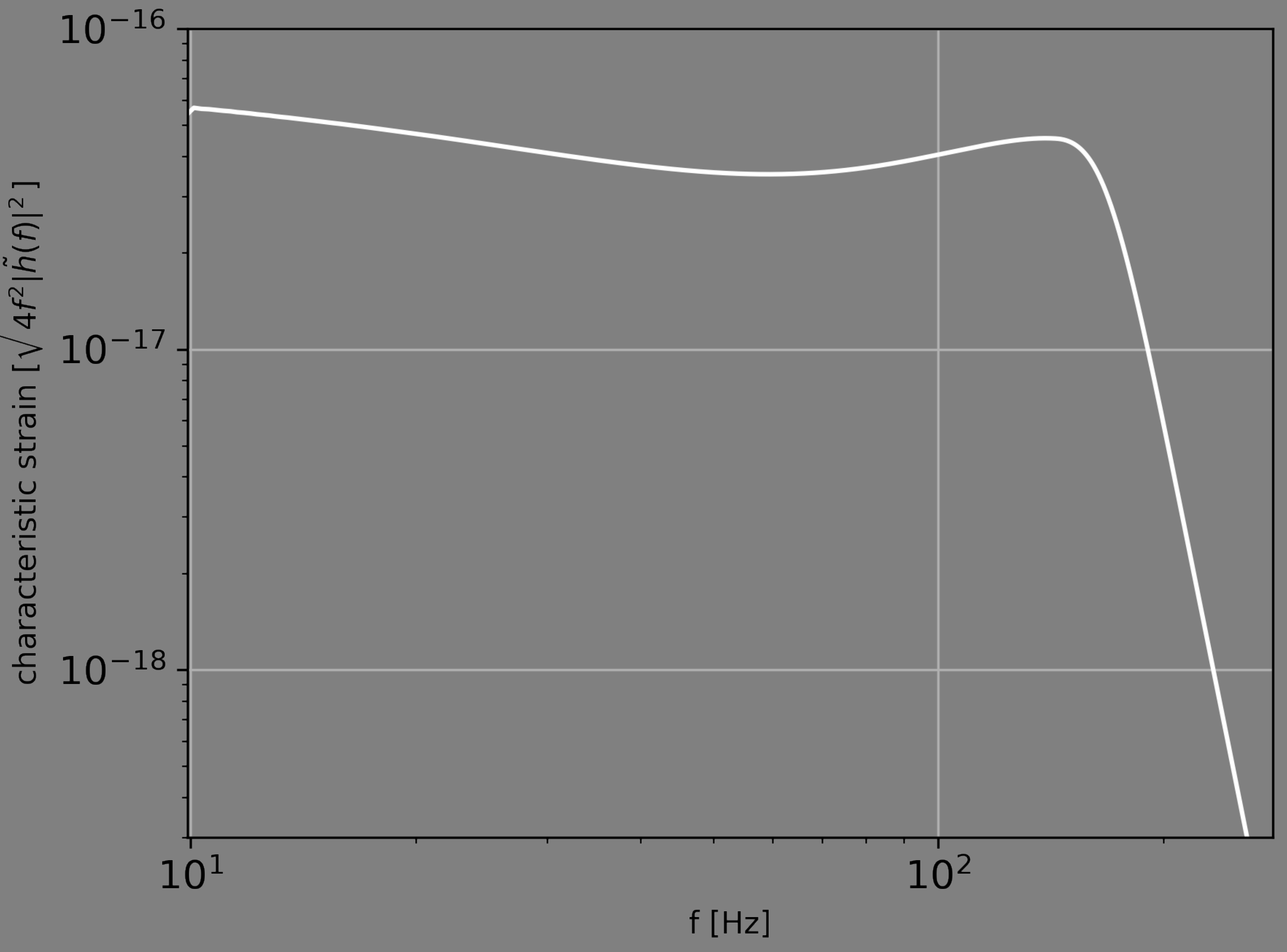
$\int$ 

$\cdot e^{-i2\pi ft} dt =$



GL of GW

$$\tilde{h}(f) \cdot F(\theta_s, f) = \tilde{h}_L(f)$$



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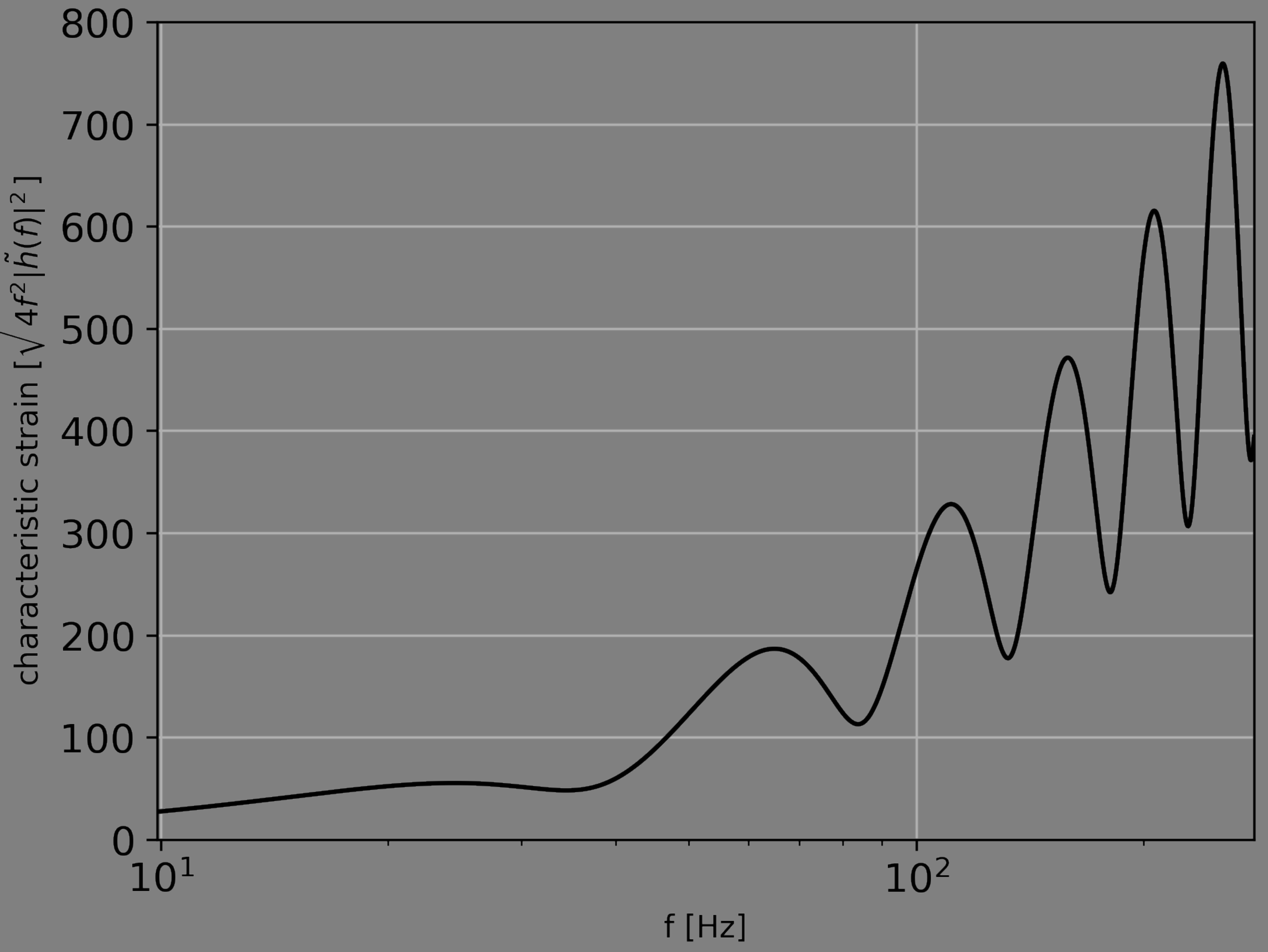

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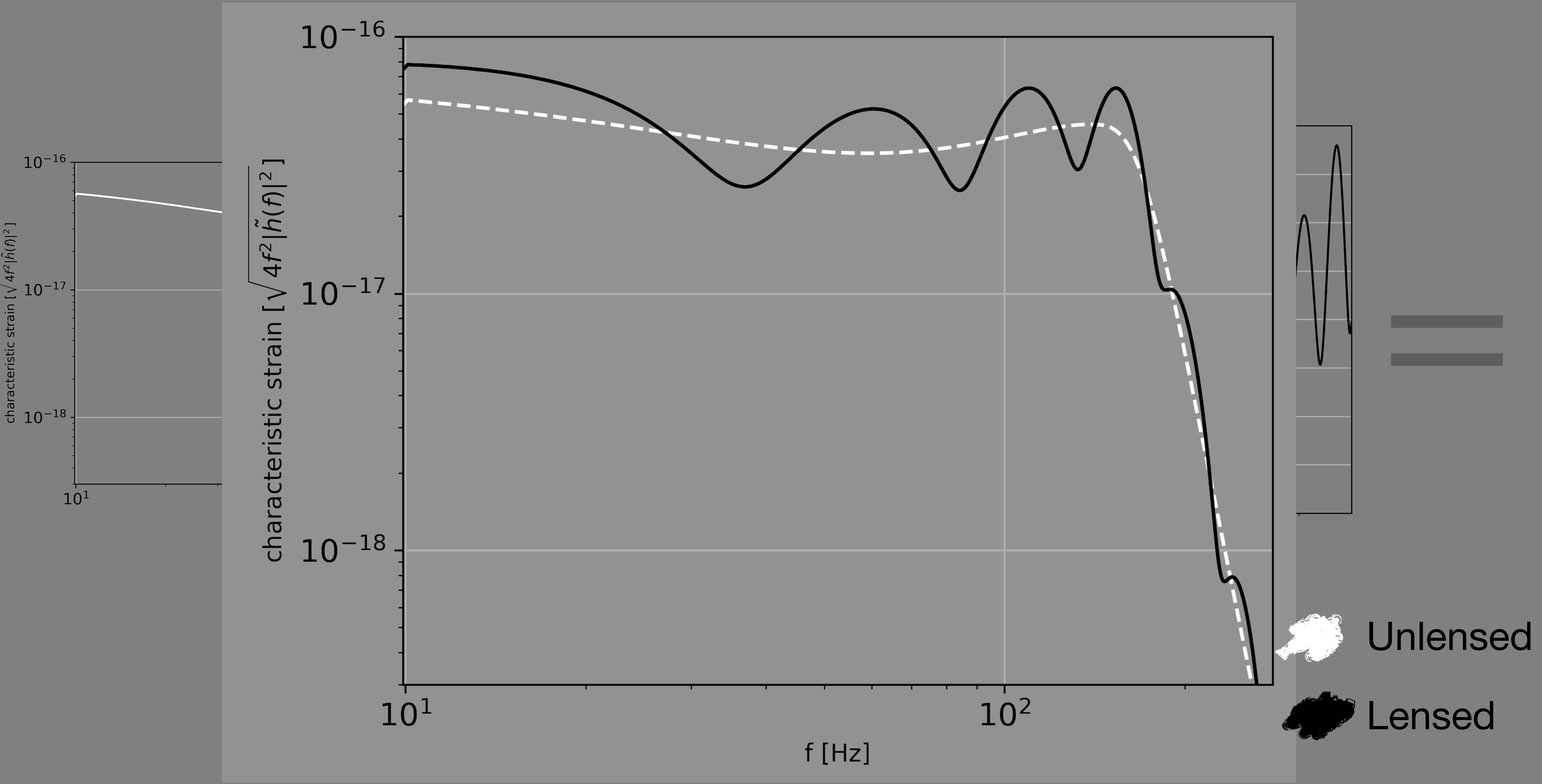
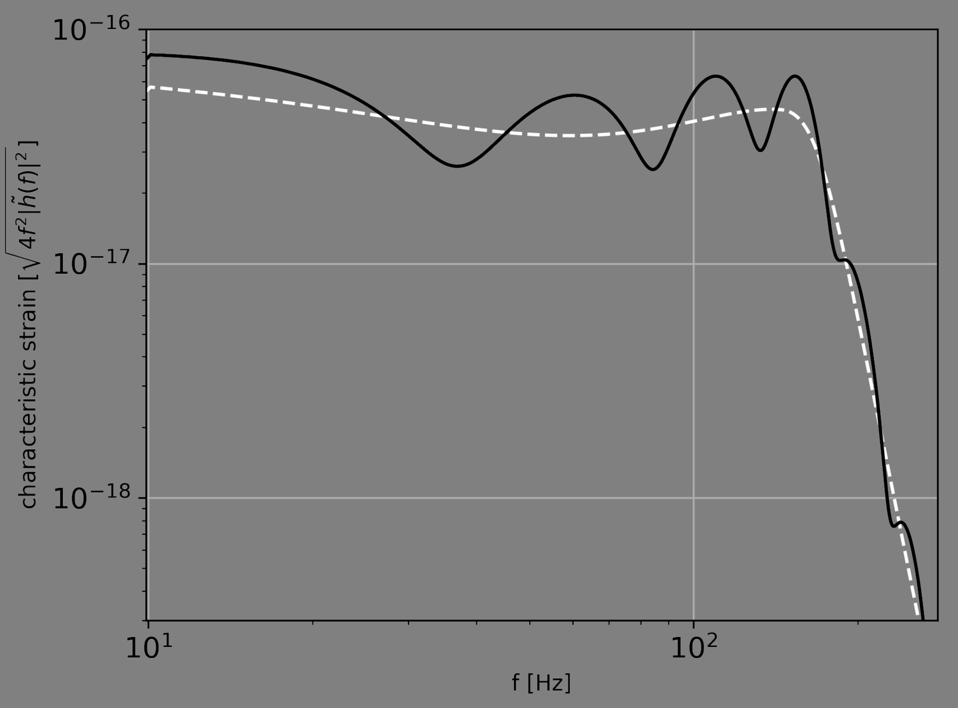
GL of GW

$$\tilde{h}(f) \cdot F(\theta_s, f) = \tilde{h}_L(f)$$

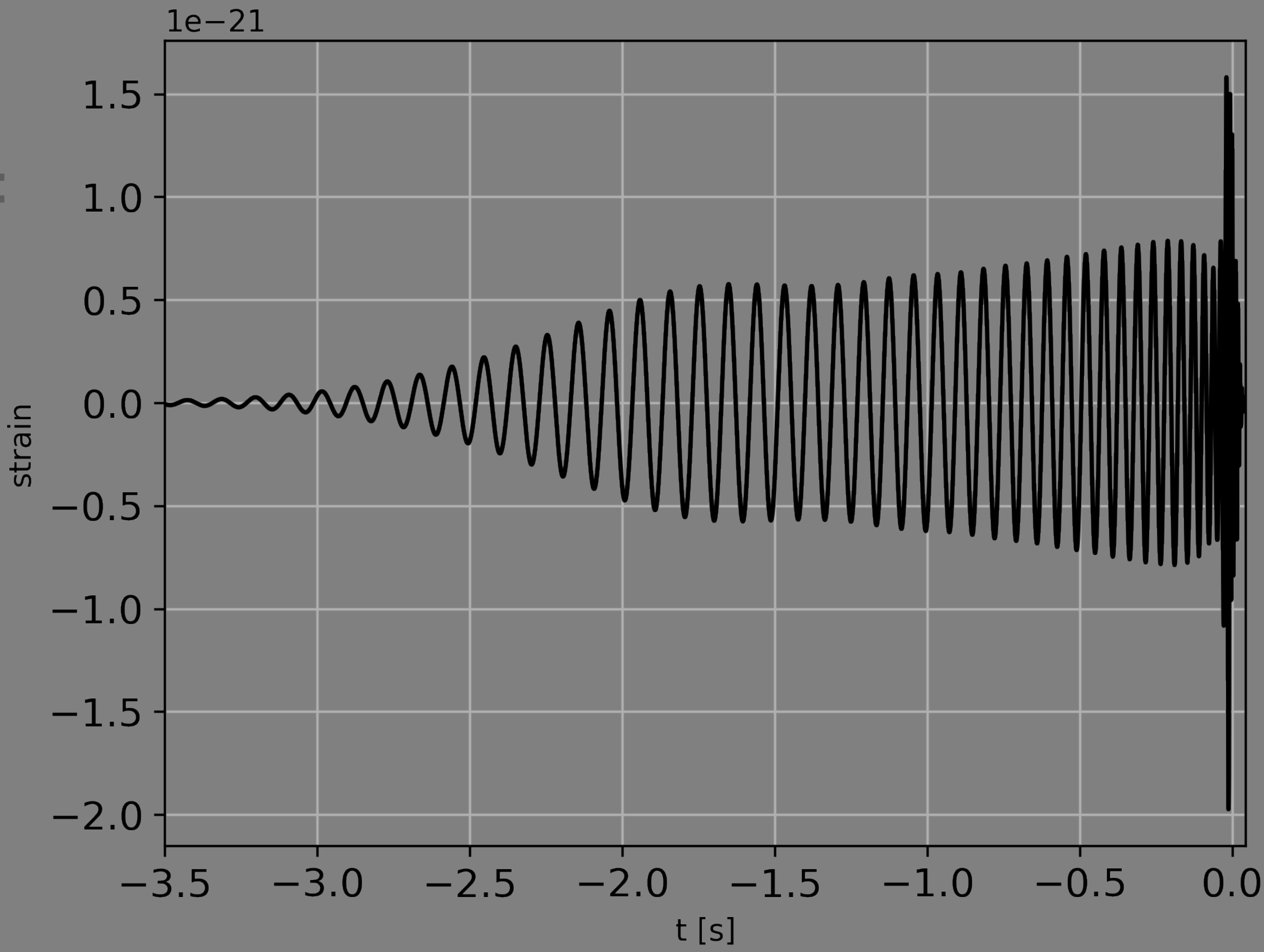


GL of GW

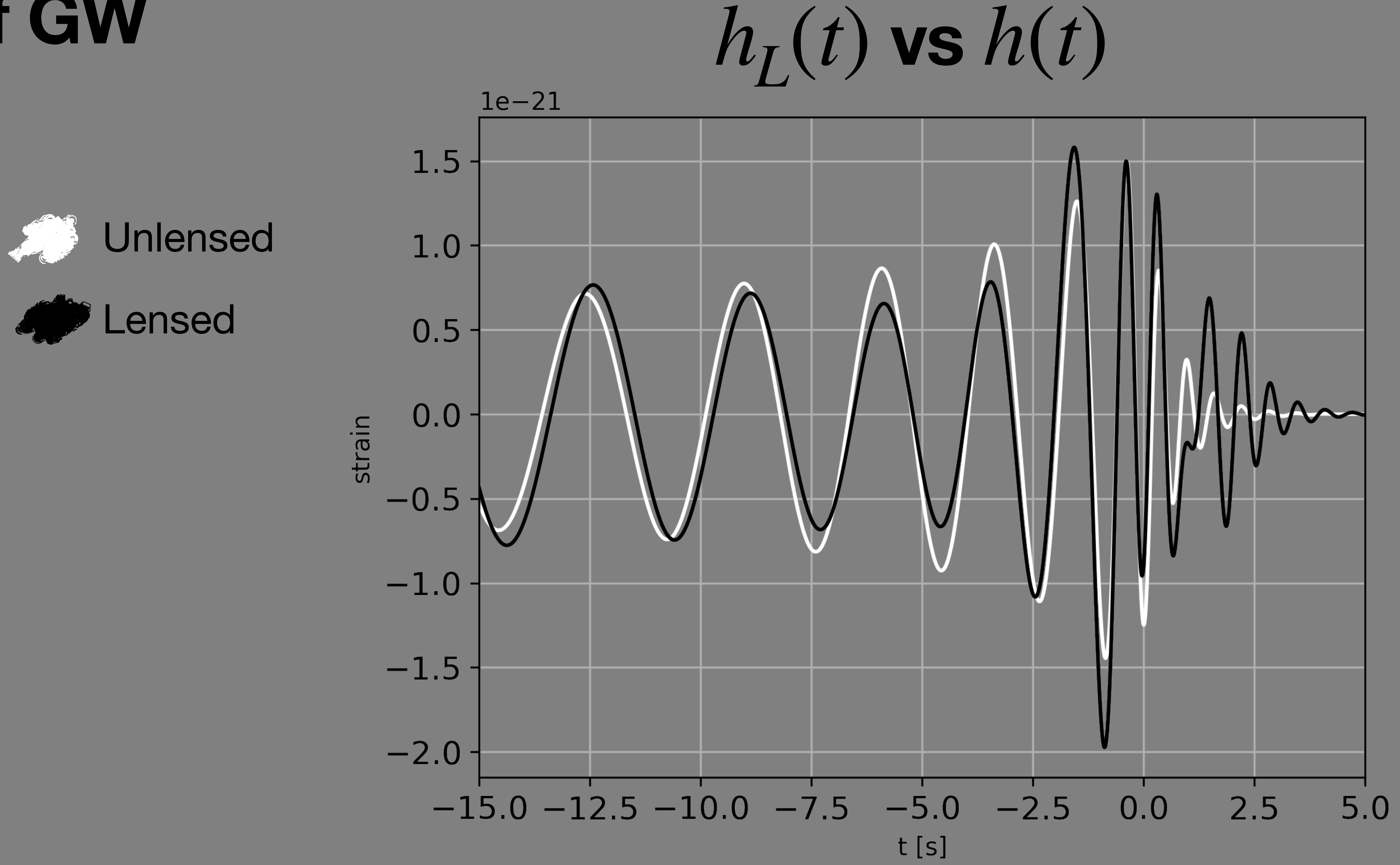
$$\int_{-\infty}^{\infty} \tilde{h}_L(f) \cdot e^{i2\pi ft} df = h_L(t)$$

$\int$ 

$\cdot\, e^{i2\pi ft} df =$



GL of GW


$h_L(t)$ vs $h(t)$
Unlensed
Lensed
1e−21
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
strain
−15.0
−12.5
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
t [s]

Amplification Factor

$$\tilde{h}(f) \cdot F(\theta_s, f) = \tilde{h}_L(f)$$

- Geometrical Optics:
  - $$F(f) = \sum_j \sqrt{\mu^{(j)}} exp(2\pi i f \Delta t^{(j)} - i n^{(j)} \pi/2)$$
- Wave Optics:
  - $$F(w, y) = -iwe^{iwy^2/2} \int_0^\infty \mathrm{d}x\, x\, J_0(wxy) \exp\left\{ iw \left[\frac{1}{2} x^2 - \Psi(x) \right] \right\}$$

- $w = \frac{1 + z_L}{c} \frac{D_S D_L \theta_E^2}{D_{LS}} 2\pi f$
- $x = |\vec{x}| = |\vec{\theta} / \vec{\theta}_E|$
- $y = |\vec{y}| = |\vec{\theta}_s / \vec{\theta}_E|$

High accuracy on H_0 constraints from gravitational wave lensing event

Based on arXiv:1911.11786 -Phys.Dark Univ. 28 (2020) 100517
with V. Salzano

Cosmology

$$F(w, y) = -\,iwe^{iwy^2/2}\int_0^\infty \mathrm{d}x\, x\, J_0(wxy)\exp\left\{iw\left[\frac{1}{2}x^2 - \Psi(x)\right]\right\}$$

- $w = \frac{1+z_L}{c}\frac{D_S D_L \theta_E^2}{D_{LS}} 2\pi f$
- $x = |\,\overrightarrow{x}\,| = |\,\overrightarrow{\theta}\,/\,\overrightarrow{\theta}_E|$
- $y = |\,\overrightarrow{y}\,| = |\,\overrightarrow{\theta}_s/\,\overrightarrow{\theta}_E|$

How?

EM-GW time-delay

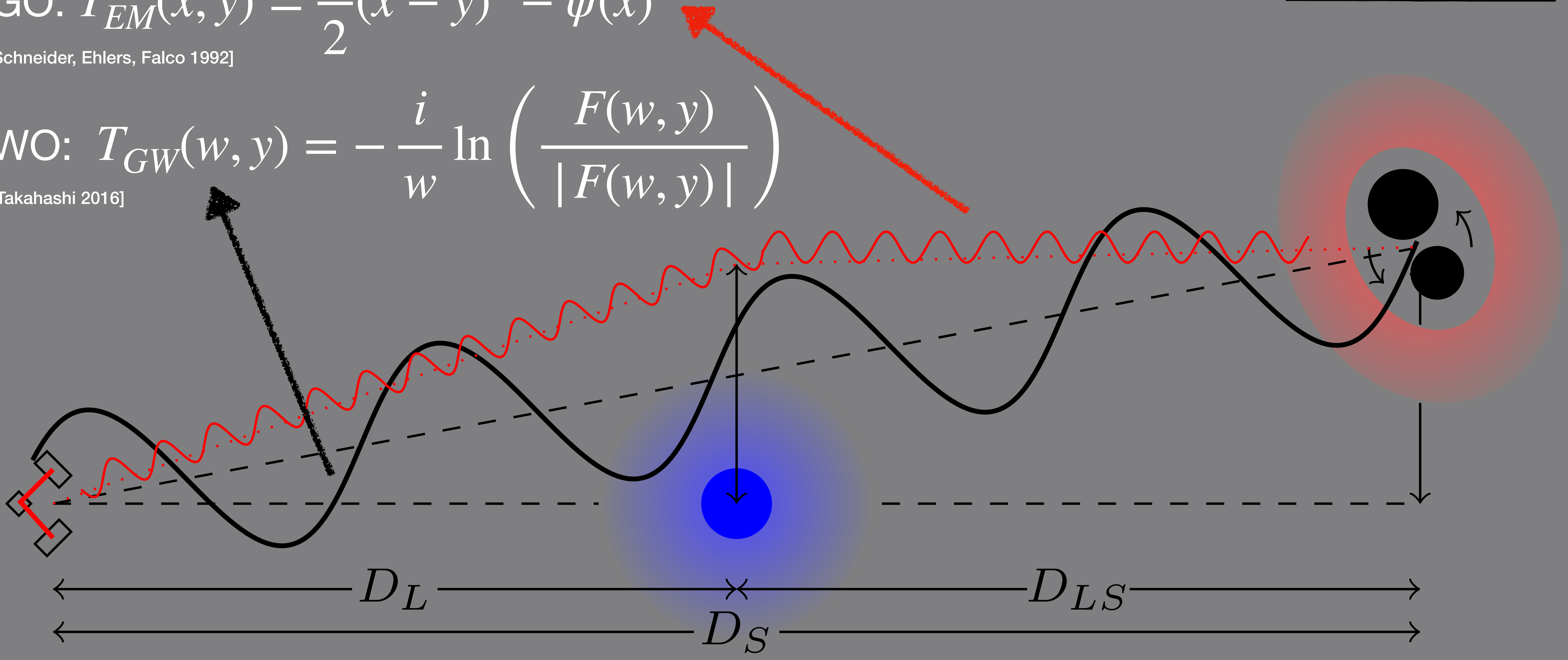
- $w = \frac{1+z_L}{c}\frac{D_S D_L \theta_E^2}{D_{LS}} 2\pi f$
- $x = |\vec{x}| = |\vec{\theta}/\vec{\theta}_E|$
- $y = |\vec{y}| = |\vec{\theta}_s/\vec{\theta}_E|$

GO: $T_{EM}(x, y) = \frac{1}{2}(x-y)^2 - \psi(x)$

[Schneider, Ehlers, Falco 1992]

WO: $T_{GW}(w, y) = -\frac{i}{w}\ln\left(\frac{F(w,y)}{|F(w,y)|}\right)$

[Takahashi 2016]



How?

EM-GW time-delay

the arrival time difference

$$T_{\mathrm{EM},\pm-\mathrm{GW}}(y,w) = T_{\mathrm{EM},\pm}(y) - T_{\mathrm{GW}}(y,w)$$

Lens Models

1. ***singular isothermal sphere (SIS)***

$$\rho(r) = \frac{\sigma_*^2}{2\pi G}\frac{1}{r^2}$$

with a stellar dispersion velocity $\sigma_*^2 = 220$ km/s

2. ***Navarro-Frenk-White (NFW)***

$$\rho(r) = \frac{\rho_0}{\frac{r}{\theta_*}\left(1 + \frac{r}{\theta_*}\right)^2}$$

assuming a realistic observed model [Buote and Barth 2019]

Methodology

- we calculate ΔT_{EM-GW} for a large set of input parameters $\{\Omega_m, H_0\}$
- we assume an independent prior on Ω_m from *Planck*, $\Omega_m = 0.3061 \pm 0.0052$
- we infer the uncertainty on H_0 by crossing the prior with the time-delay uncertainty

Methodology

uncertainty on GW time-delay

$$\sigma_{\Delta T} = (2\pi f \rho^2)^{-1}$$

where

[Huerta at el. 2015]

$$\rho^2 = \hat{\rho}^2 \cdot (1+z)^4 \left(\frac{f_{\rm orb}}{f_{\rm obs}} \right)^{-2/3}$$

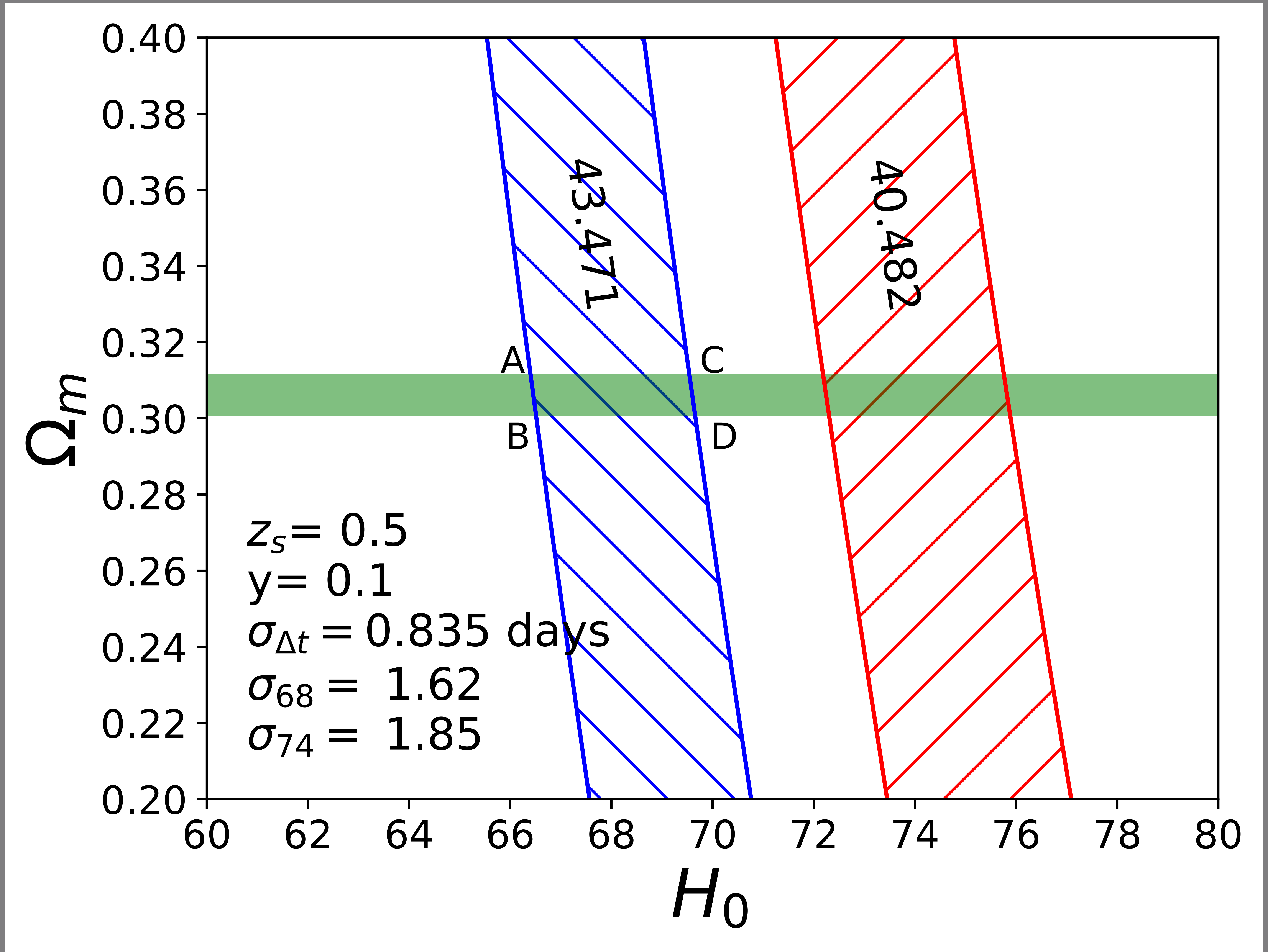
Methodology

- state-of-the-art sample made of 65 pulsars observed with PTA [Perera at el. 2019]
- an "optimistic" future sample of 1000 pulsars detected with SKA [Weltman at el. 2018]

z_S	$\mathcal{M}$ ($10^8 M_\odot$)	T_{obs} (yr)	σ_{rms} (ns)	$\Delta\tau$ (yr)	N_p	$\sigma_{\Delta T}$ (days)
0.5	5	10	100	0.038	65	0.835
					1000	0.003
1					65	1.431
					1000	0.006

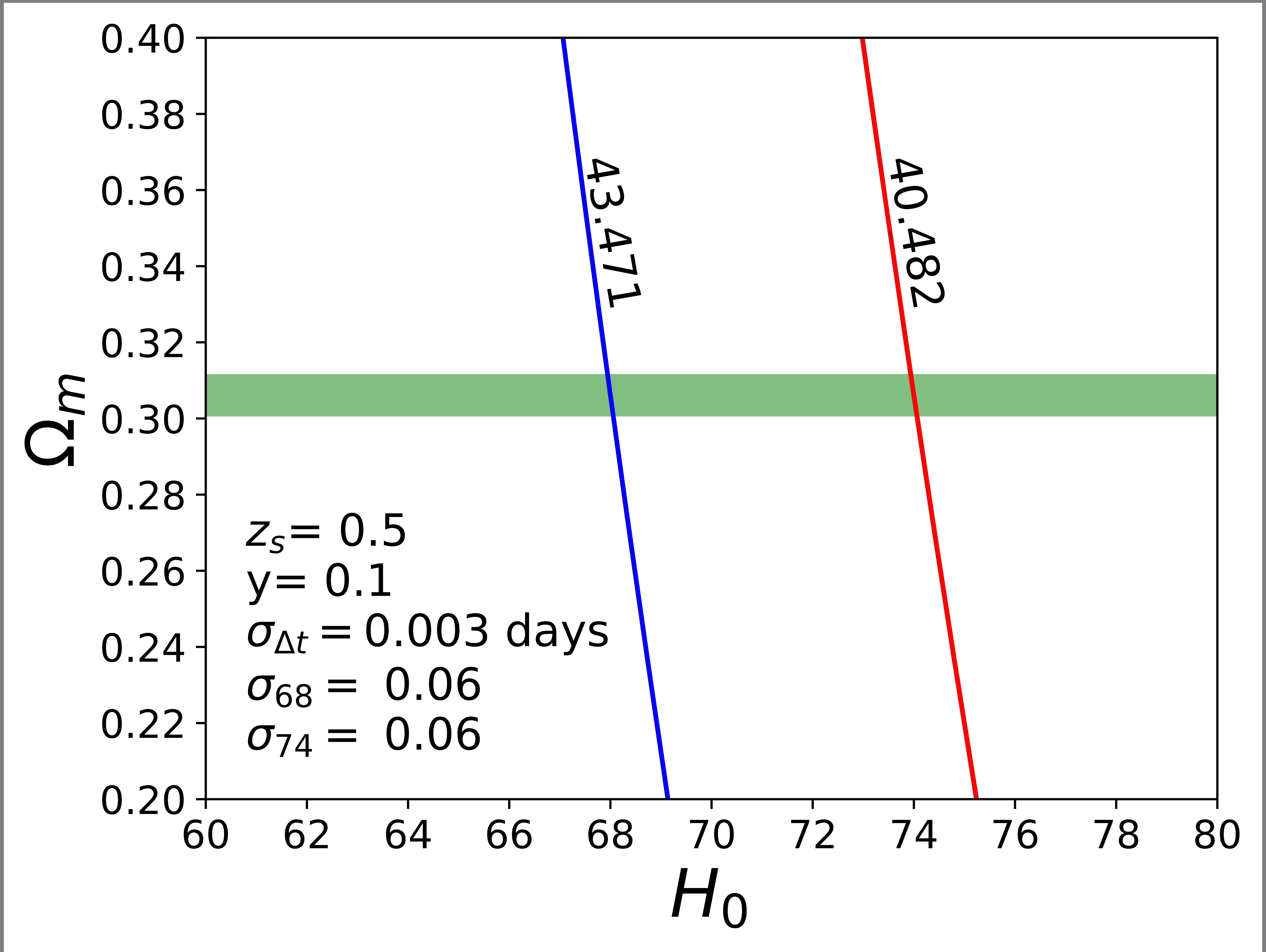
Results

NFW lens - IPTA 65 pulsars array



Results

NFW lens - SKA 1000 pulsars array



Results

z_S	0.5				1			
$\sigma_{\Delta T}$ (days)	0.835		0.003		1.431		0.006	
H_0 (km s^{-1} Mpc^{-1})	68	74	68	74	68	74	68	74
y ↓	NFW - ΛCDM							
0	1.37	1.55	0.06	0.06	2.19	2.47	0.06	0.07
0.1	1.62	1.85	0.06	0.06	2.60	2.94	0.06	0.07
0.5	3.72	4.49	0.06	0.07	5.60	6.05	0.07	0.08
	NFW - quiessence							
0	14.50	15.80	12.10	14.20	14.70	16.20	12.60	13.70
0.1	14.60	16.00	12.60	14.20	15.00	16.70	12.60	13.70
0.5	16.50	18.20	13.10	14.30	16.90	19.30	12.70	13.90
	SIS - $\sigma_* = 220$ (km/s) - ΛCDM							
0	2.80	3.15	0.06	0.07	4.56	5.13	0.07	0.08
0.1	3.06	3.43	0.06	0.07	5.03	5.63	0.07	0.08
1	>10	9.70	0.10	0.10	>10	>10	0.17	0.16
	SIS - $\sigma_* = 220$ km/s - quiessence							
0	15.80	17.30	12.90	14.20	16.10	18.70	12.70	13.90
0.1	16.00	17.60	12.90	14.20	16.40	19.00	12.70	13.90
1	>20.00	>20.00	13.20	14.40	>20.00	>20.00	12.80	13.90

$$H^2(z) = H_0^2 \cdot [\Omega_\gamma(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda(1+z)^{3(1+w)}]$$

Conclusions 1/3

- need for different measurement to decrease the error ($\sigma \sim 1/\sqrt{n}$)
- today observations could match current precision on H_0
- SKA will give decisive results

Mass Sheet Degeneracy

Based on arXiv:2104.07055 - Phys. Rev. D 104, 023503 (2021)

with J.M. Ezquiaga and V. Salzano

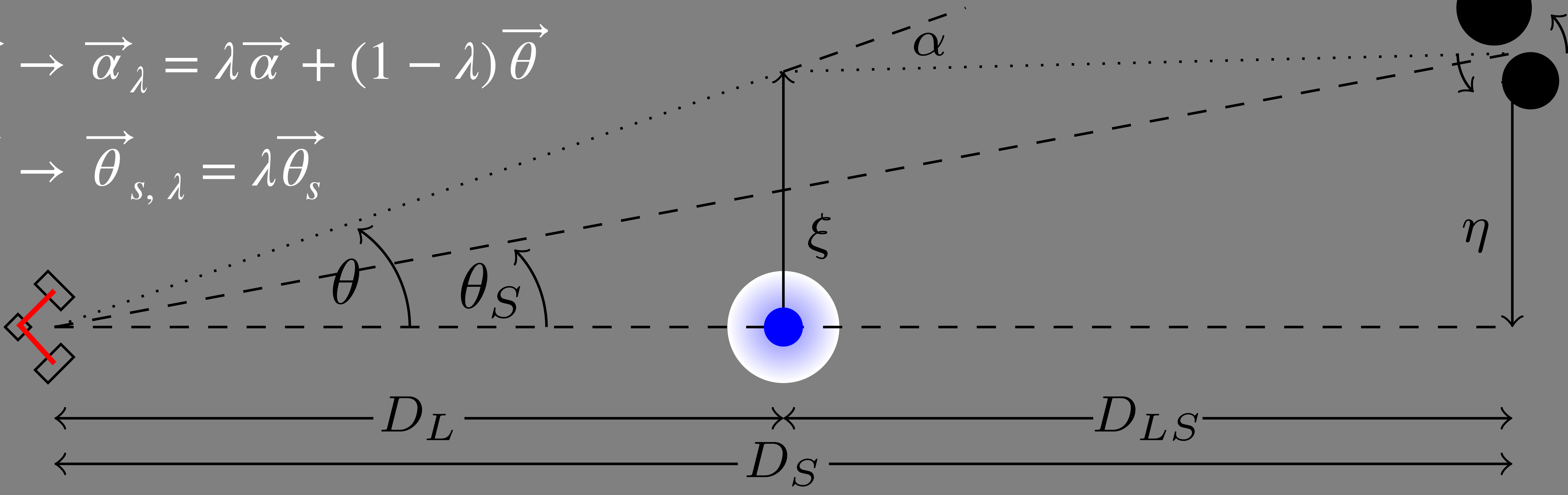
Mass Sheet Degeneracy

E. E. Falco, M. V. Gorenstein, and I. I. Shapiro, ApJ 289, L1 (1985)

- Scalings of lens mass:
  - $\kappa \rightarrow \kappa_\lambda = \lambda\kappa + (1 - \lambda)$
- Scaling angles:
  - $\vec{\alpha} \rightarrow \vec{\alpha}_\lambda = \lambda\vec{\alpha} + (1 - \lambda)\,\vec{\theta}$
  - $\vec{\theta}_s \rightarrow \vec{\theta}_{s,\,\lambda} = \lambda\vec{\theta}_s$

$\kappa = \Sigma/\Sigma_{cr}$

Σ - surface mass density



MSD

Why a problem?

- Observables are preserved!
- Problems: e.g. biased estimations of mass lens
- Biased estimation of cosmological parameter, e.g. H_0

Can we solve it?

- EM geometrical optics regime: multiple images; independent mass estimation of the lens (e.g. dynamics)
- EM wave optics regime: multiple lenses
- **In GW lensing: 1 image and 1 lens can break MSD!**

Gravitational Lensing of Grav. Waves

NB: spherical symmetry!

- $\tilde{h}(f) \cdot F(f, \theta_s) = \tilde{h}_L(f)$
- $F(w, y) = -iwe^{iwy^2/2} \int_0^\infty \mathrm{d}x\, x\, J_0(wxy) \exp\left\{ iw \left[\frac{1}{2} x^2 - \Psi(x) \right] \right\} \rightarrow F_\lambda$

T. T. Nakamura and S. Deguchi, Progress of TheoreticalPhysics Supplement133, 137 (1999).

- Where:
  - $w = \frac{1 + z_L}{c} \frac{D_S D_L \theta_E^2}{D_{LS}} 2\pi f$
  - $x = |\vec{x}| = |\vec{\theta} / \vec{\theta}_E|$
  - $y = |\vec{y}| = |\vec{\theta}_s / \vec{\theta}_E| \rightarrow y_\lambda$

- J_0 - Bessel function of 0-th order
- Ψ - dimensionless effective lensing potential $\rightarrow \Psi_\lambda$

Lensed waveforms under mass-sheet transformation

Qualitative analysis

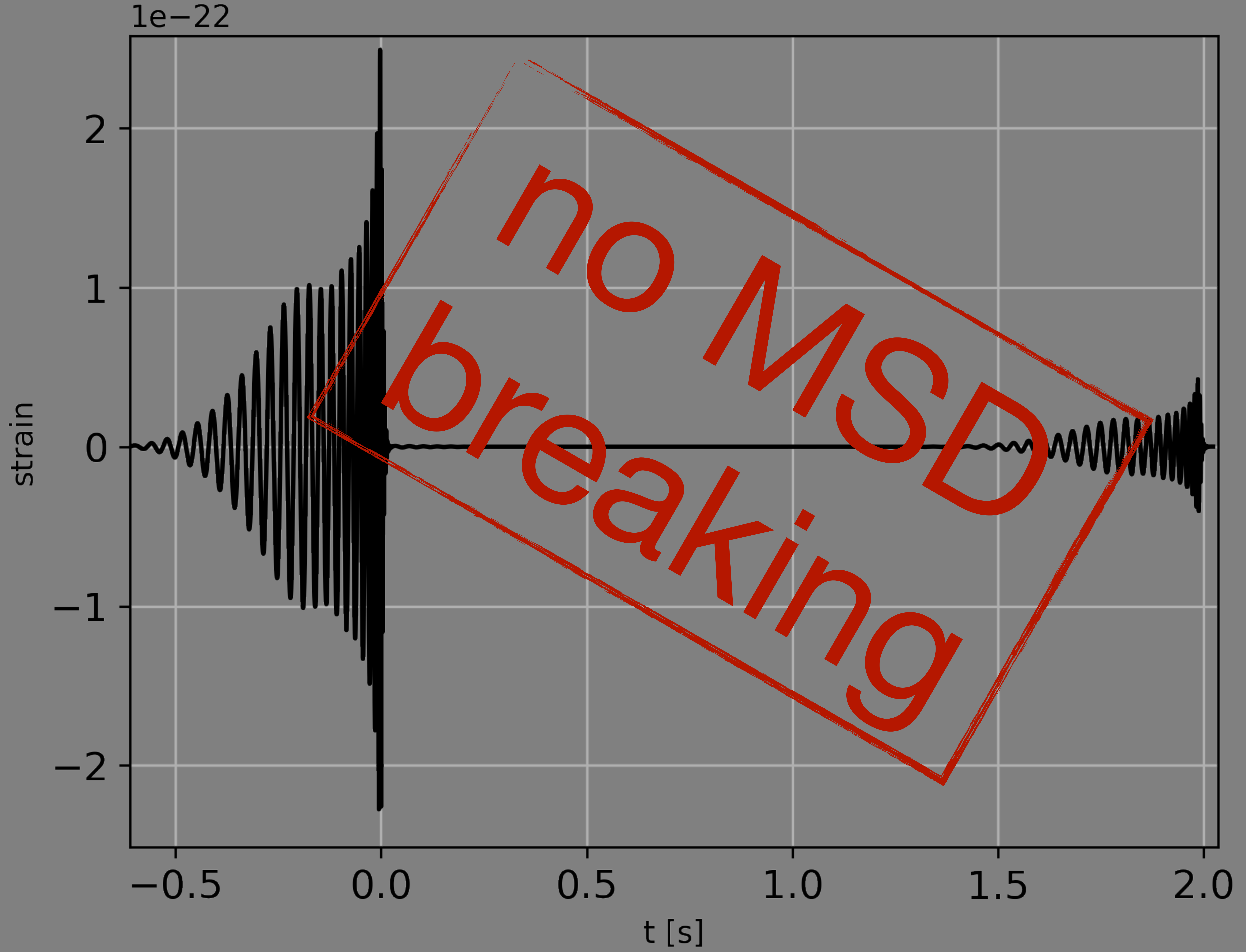
Lensed GWs

3 regimes

- Geometrical Optics
  - $f \cdot \Delta t \gg 1$
  - $M_L > 10^5[(1+z_L)f]^{-1}$

$M_S = 60\ M_\odot\ -\ z_S = 0.5$

$M_L = 10^4\ M_\odot\ -\ z_L = 0.1\ -\ y = 5$



R.Takahashi,Astrophys.J.835,103(2017),arXiv:1606.00458 [astro-ph.CO].

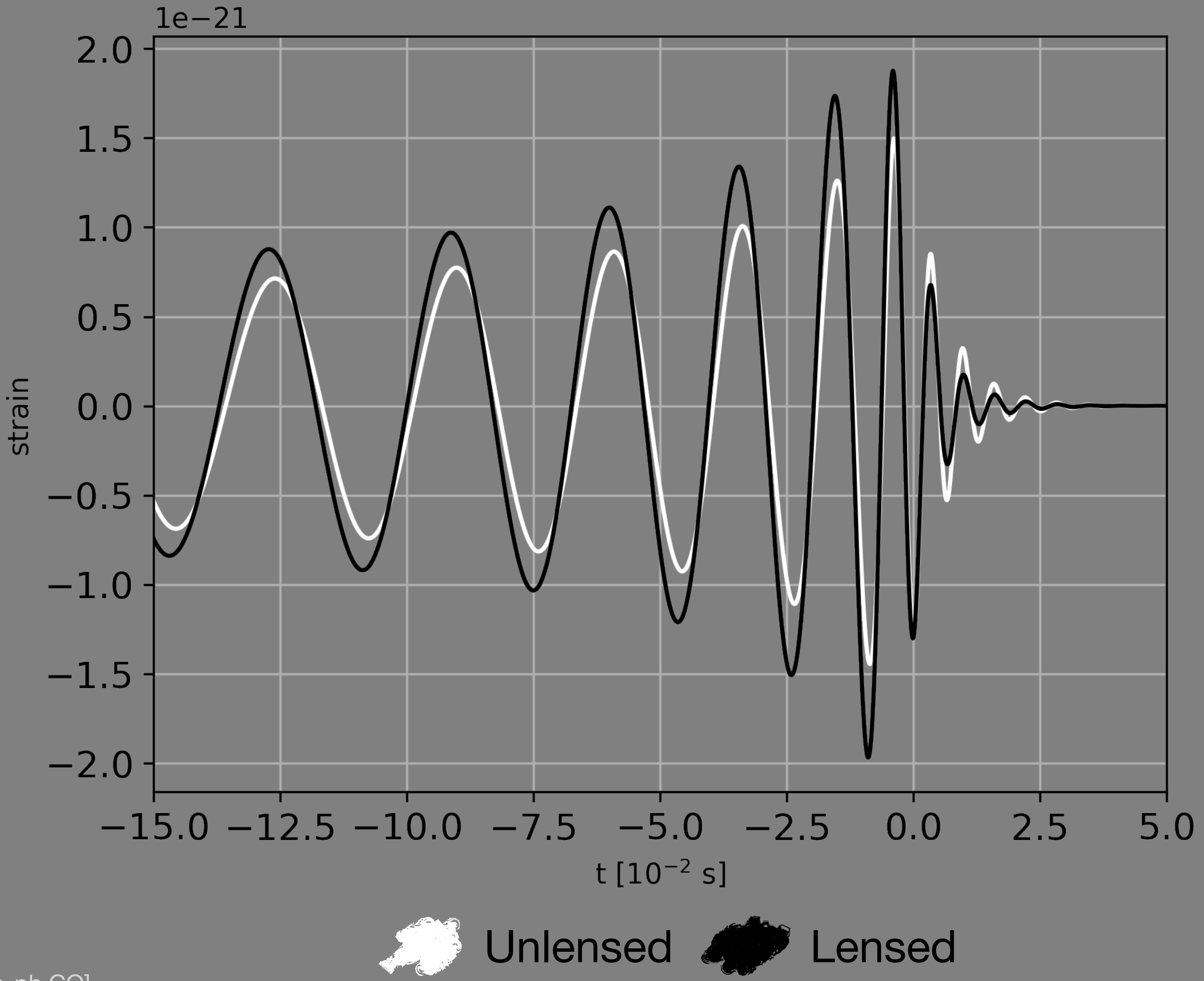
Lensed GWs

3 regimes

- Wave Optics
  - $f \cdot \Delta t \lesssim 1$
  - $M_L \leq 10^5[(1+z_L)f]^{-1}$

$M_S = 100\ M_\odot\ -\ z_S = 0.1$

$M_L = 100\ M_\odot\ -\ z_L = 0.01$



R.Takahashi,Astrophys.J.835,103(2017),arXiv:1606.00458 [astro-ph.CO].

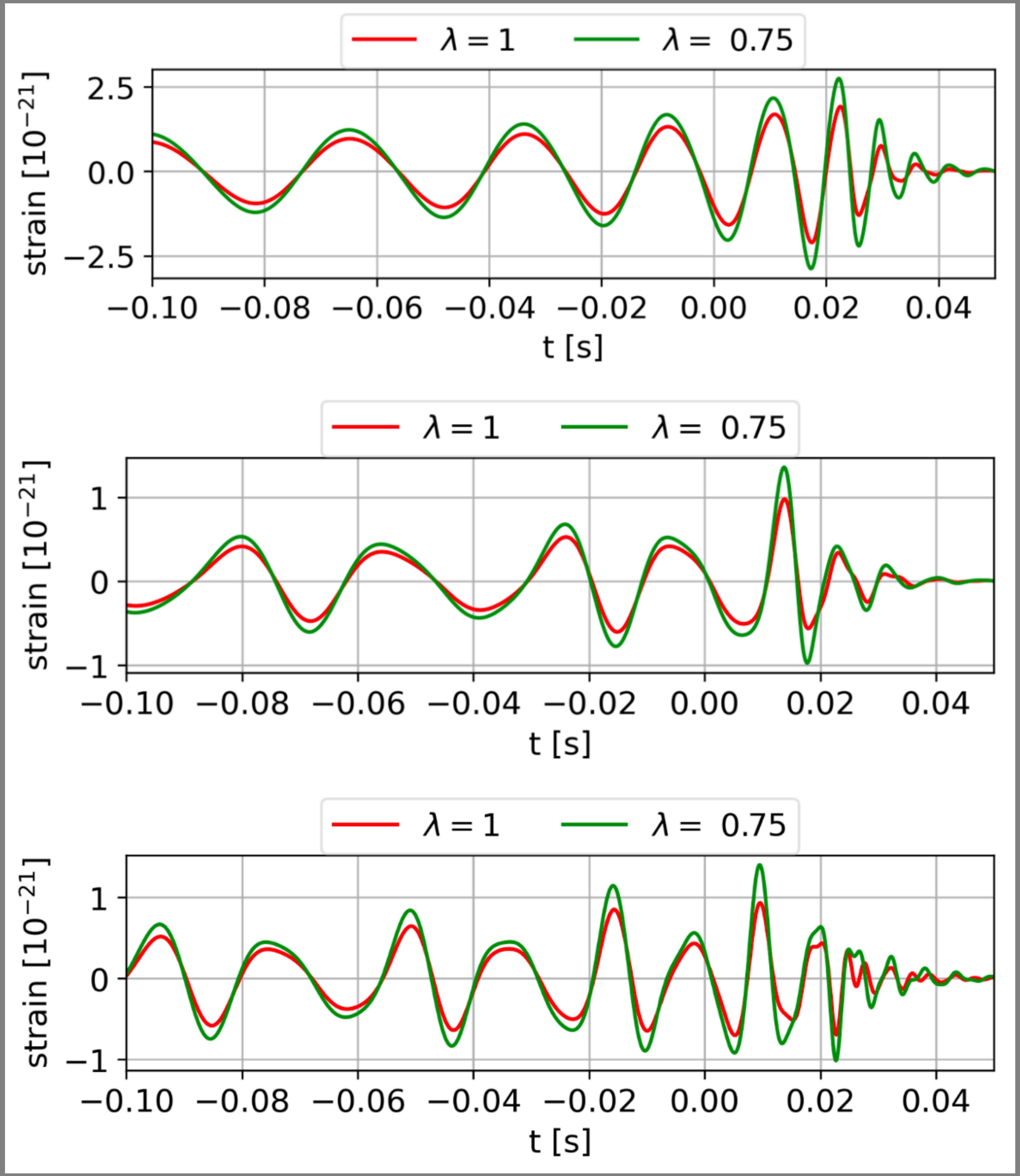
Lensed GWs

Wave optics

$$q = \frac{m_2}{m_1} = 1$$

$$q = 0.1$$

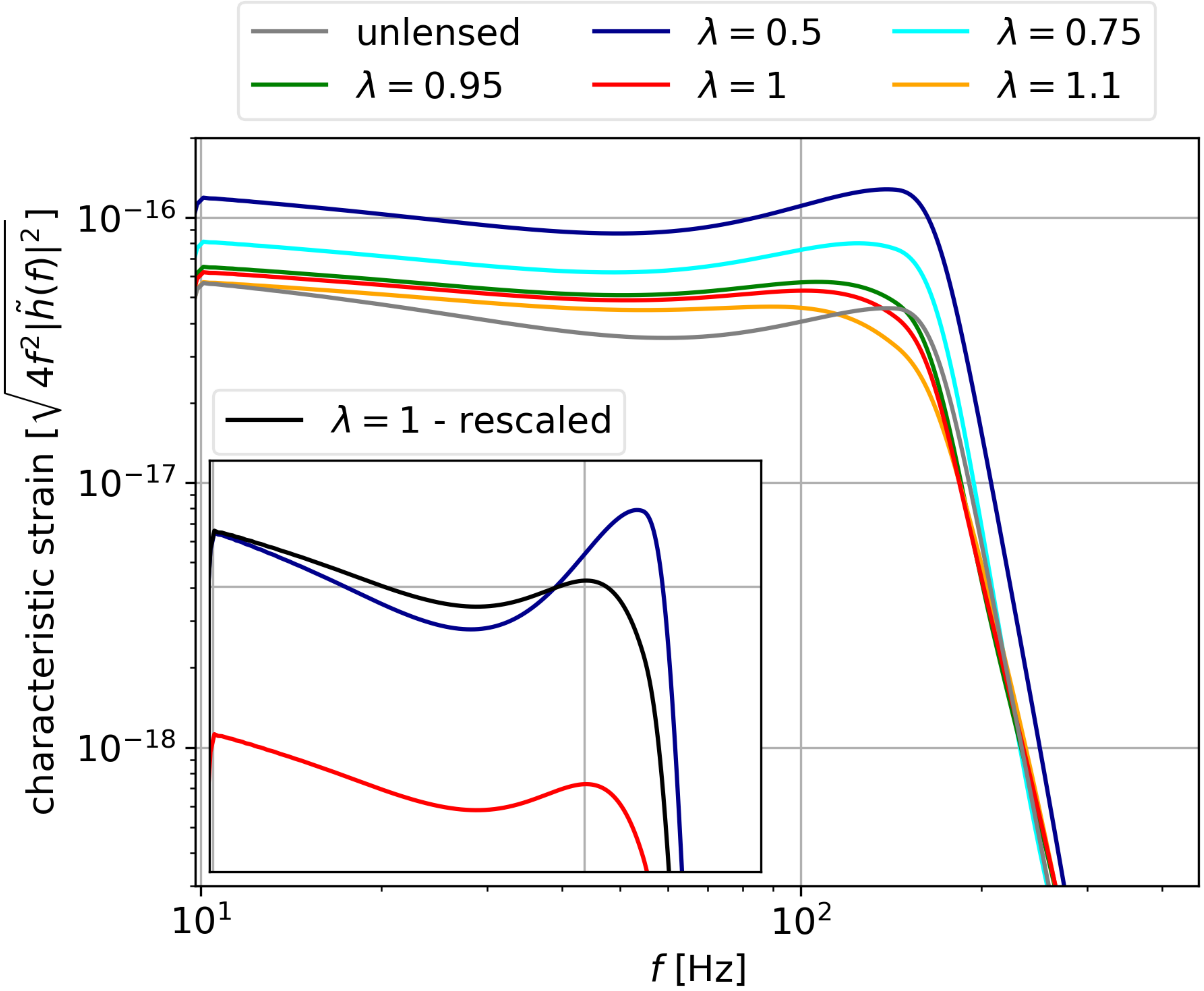
$$q = 0.1 \ \& \ s_{1,2;z} = \{0.7, 0.2\}$$



Lensed GWs

Wave optics

$q = 1$



Lensed GWs

3 regimes

Interference regime: $f \cdot \Delta t \approx 1$

$M_L = 500\ M_\odot$ & $y = 1$ & $z_L = 0.01$

$M_S = 100\ M_\odot$ & $q = 1$ & $z_S = 0.1$

Lensed GWs

3 regimes

Interference regime: $f \cdot \Delta t \approx 1$

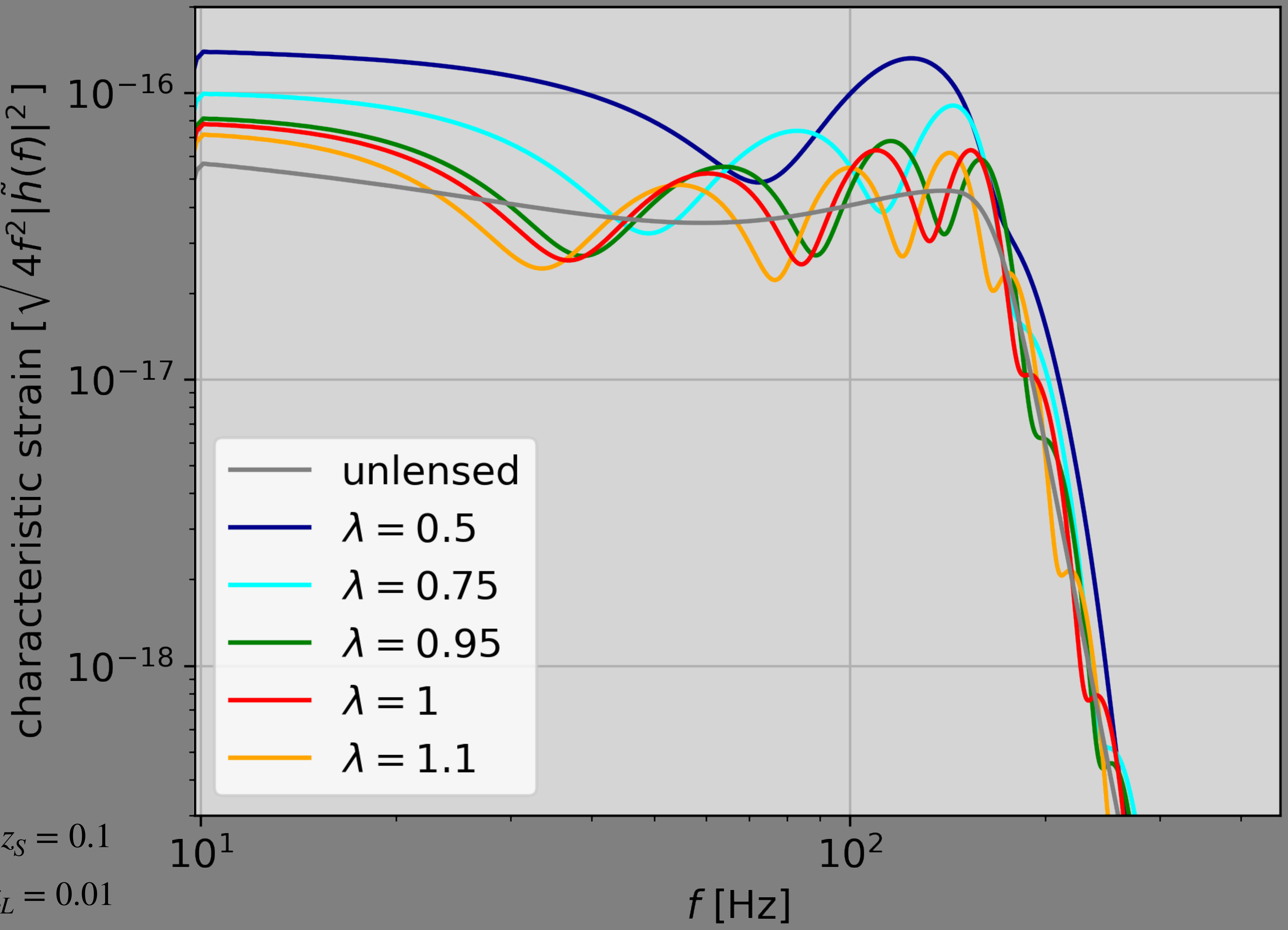
$M_L = 500\ M_\odot$ & $y = 1$ & $z_L = 0.01$

$M_S = 100\ M_\odot$ & $q = 1$ & $z_S = 0.5$

Lensed GWs

Interference regime

- $f \cdot \Delta t \approx 1$



$M_S = 100\ M_\odot\ \&\ q = 1\ \&\ z_S = 0.1$

$M_L = 500\ M_\odot\ \&\ y = 1\ \&\ z_L = 0.01$

S/N - template matching

Quantitative analysis

Signal-to-Noise ratio

$$\rho = \frac{(s|h_T)}{\sqrt{(h_T|h_T)}} \approx \frac{(h|h_T)}{\sqrt{(h_T|h_T)}}$$

- $s(t) = h(t) + n(t)$
- Inner product:

$$(a|b) = 4\,\mathrm{Re}\left[\int_0^\infty \frac{\tilde{a}(f)\cdot\tilde{b}^*(f)}{S_n(f)}df\right]$$

  - $S_n(f)$ - (single-sided) power spectral density (L1-O3-LIGO)

Confidence region: $$\Delta\chi^2 \approx 2\rho_{opt}^2\left[1 - \frac{\rho}{\rho_{opt}}\right]$$

$$3\sigma \rightarrow \Delta\chi^2 \approx 11.8$$

M. Maggiore,Gravitational Waves: Volume 1: Theory and Experiments,Gravitational Waves (OUP Oxford, 2008)

S/N

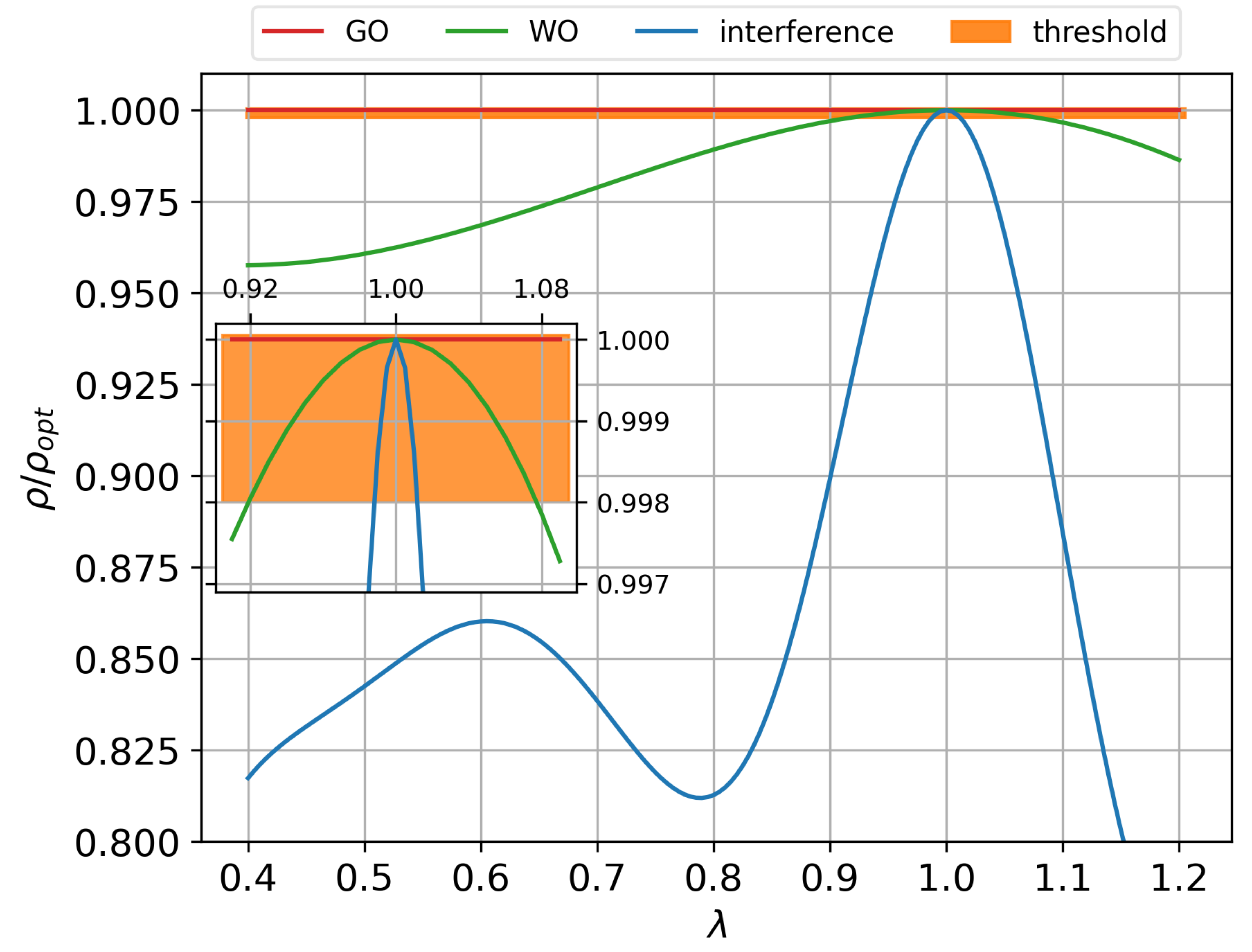
- $M_S = 100\ M_\odot$
- $z_S = 0.1$
- $z_L = 0.01$

$3\sigma \rightarrow \Delta\chi^2 \approx 0.998$

GO $M_L = 500 M_\odot$ $y = 10$

Int. $M_L = 500 M_\odot$ $y = 1$

WO $M_L = 100 M_\odot$ $y = 1$



S/N
$\lambda_{min} = 0.93$
$\lambda = 1$
$\lambda_{max} = 1.03$
Interference regime
Signal
$M_L = 500$
$y = 1$
$z_S = 0.5$
$\rho_{opt} \approx 11$
$\rho_{opt} \approx 55$
Signal
$M_L = 500$
$y = 1$
$z_S = 0.1$
$\lambda_{min} = 0.99$
$\lambda = 1$
$\lambda_{max} = 1.01$
M_L [10^2 $M_\odot$]
y
ρ/ρ_{opt}

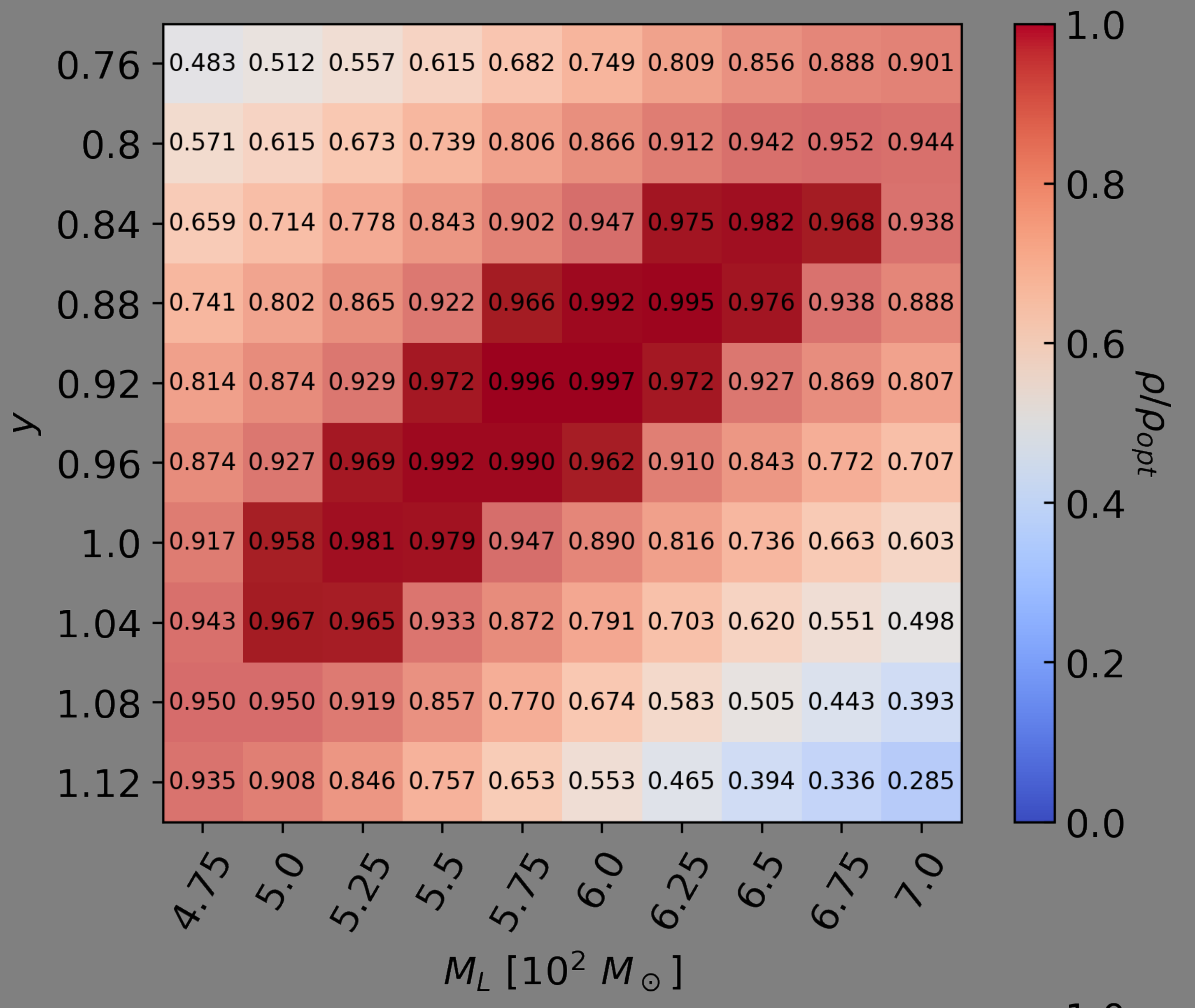
S/N

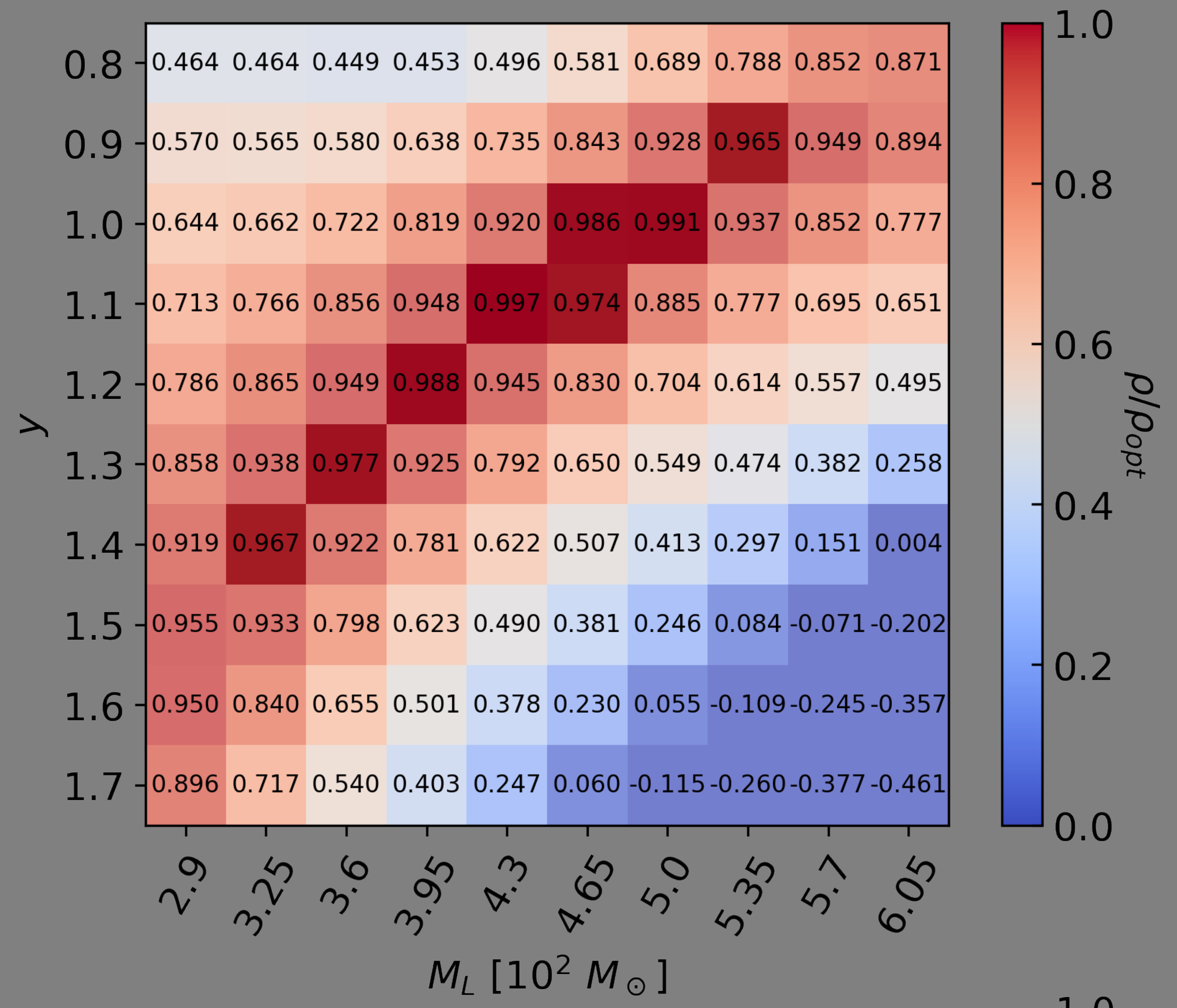
Interference regime

$\lambda_{min} = 0.93$

$\lambda_{max} = 1.03$

$\rho_{opt} \approx 11$





$\Delta y < 40\,\%$

$\Delta M_L \approx 35\,\%$

$>$

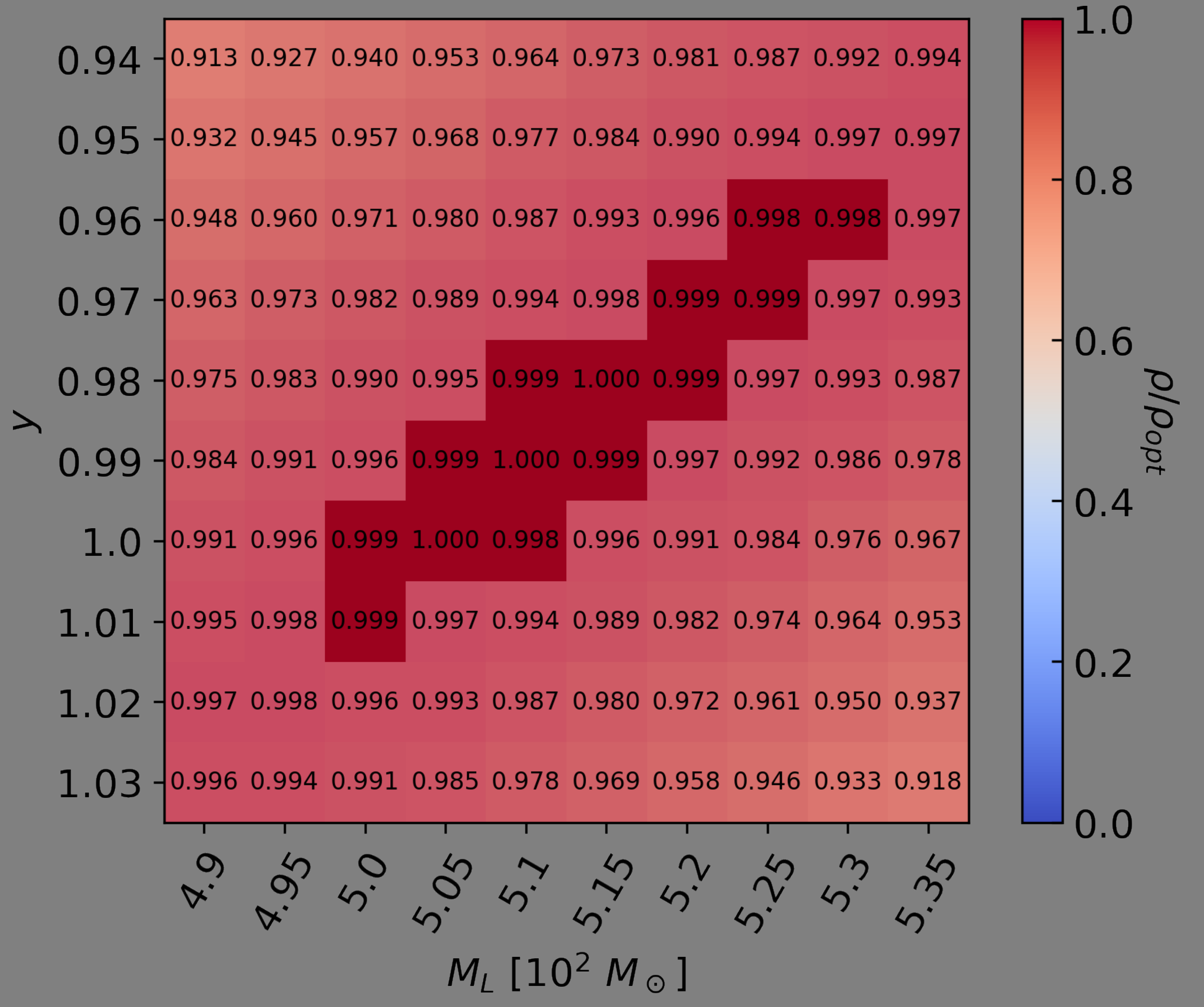
$\Delta M_L \approx 12 - 20\,\%$

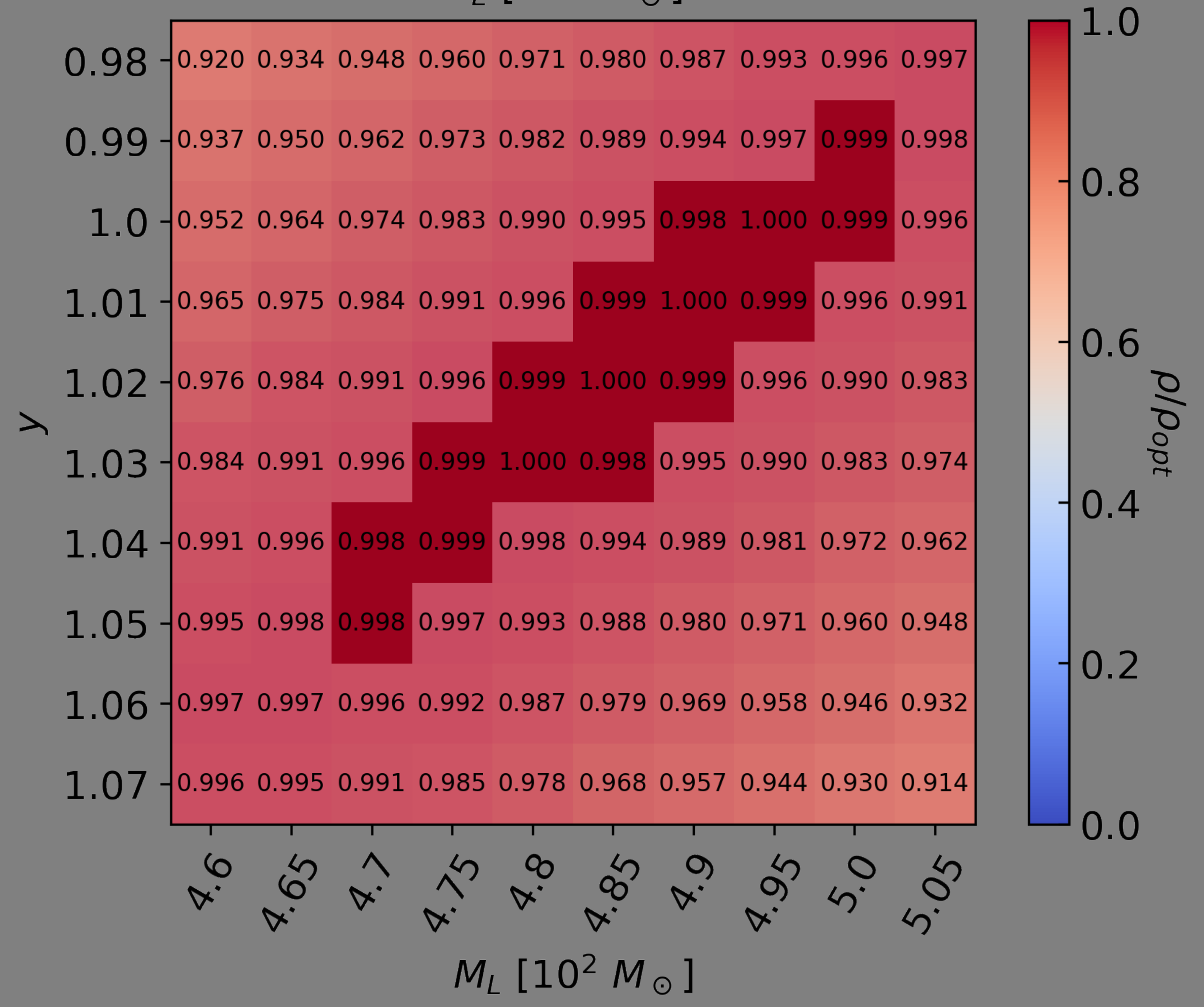
$>$

$\Delta y \approx 5\,\%$

$\Delta M_L \approx 6\,\%$

$\rho_{opt} \approx 55$





$\lambda_{min} = 0.99$

$\lambda_{max} = 1.01$

P. Schneider and D. Sluse, Astron. Astrophys.559, A37(2013), arXiv:1306.0901 [astro-ph.CO].

Conclusions 2/3

Conclusions 2/3

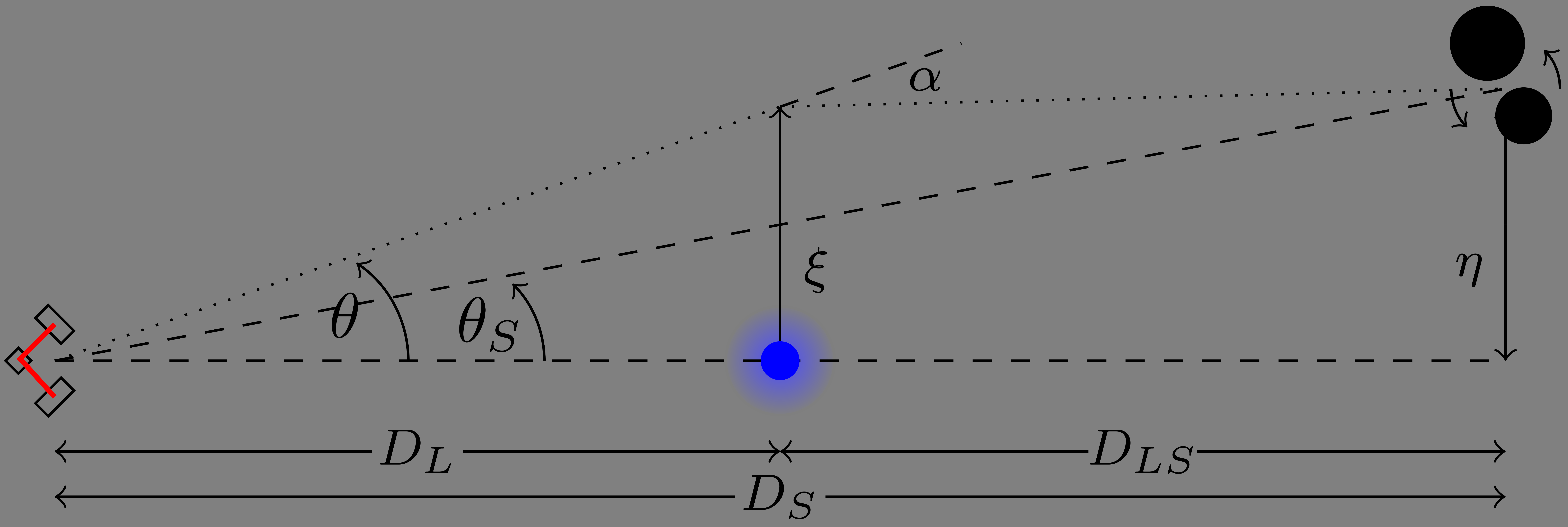
1. We analysed how MSD behave in GW lensing
2. In the GO regime it can not be broken
3. In WO can be broken in some cases
4. In interference regime is broken
5. How well it is broken depends on the strength of the signal and sensitivity of detectors. Nowadays we might have up to $\Delta y \approx 5\,\%$ and $\Delta M \approx 6\,\%$

High precision lens modelling

Based on arXiv:2111.01163

with D.F. Mota and V. Salzano

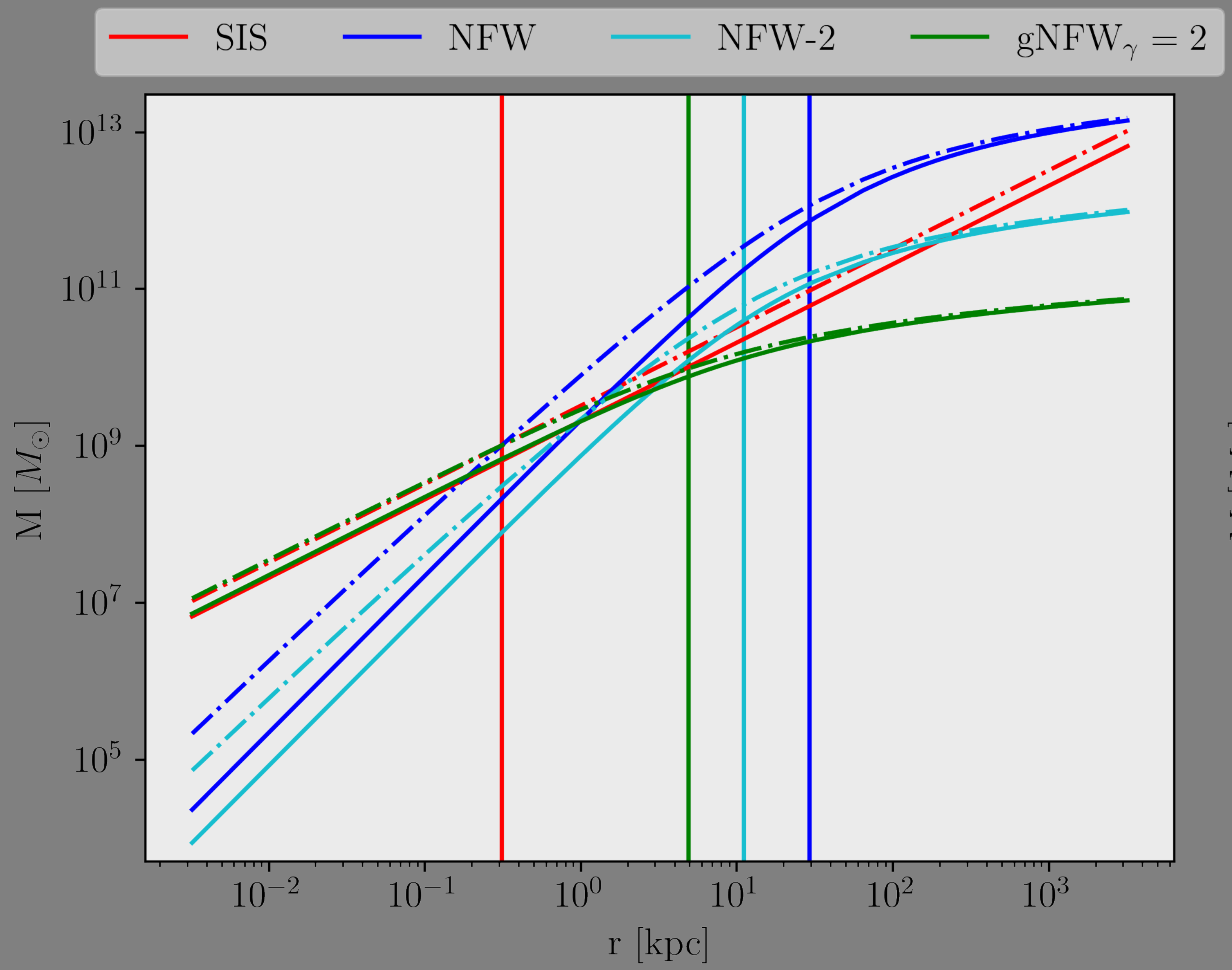
Gravitational Wave lensing



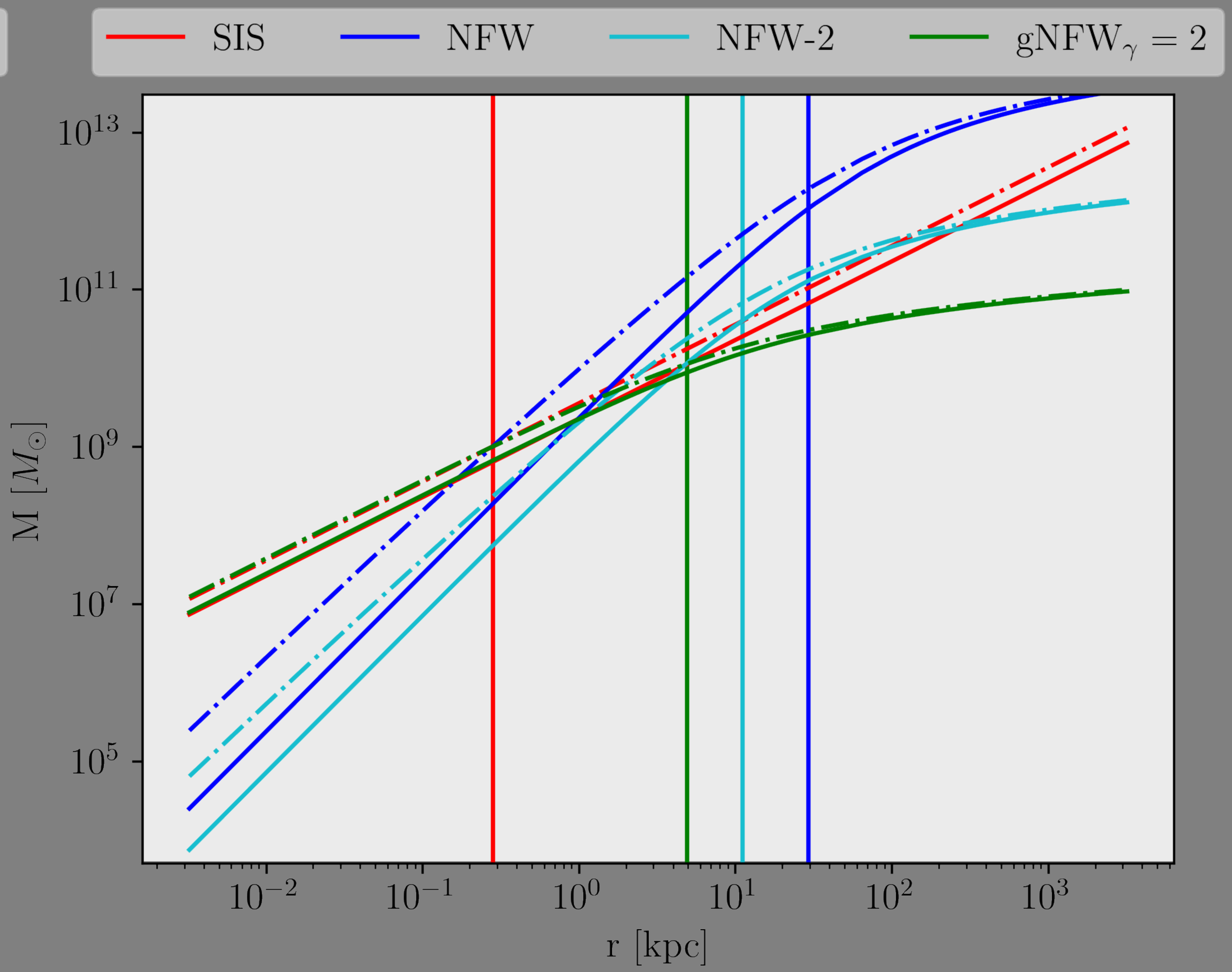
High precision lens modelling

Lens mass profile

$z_L = 0.5$



$z_L = 0.15$



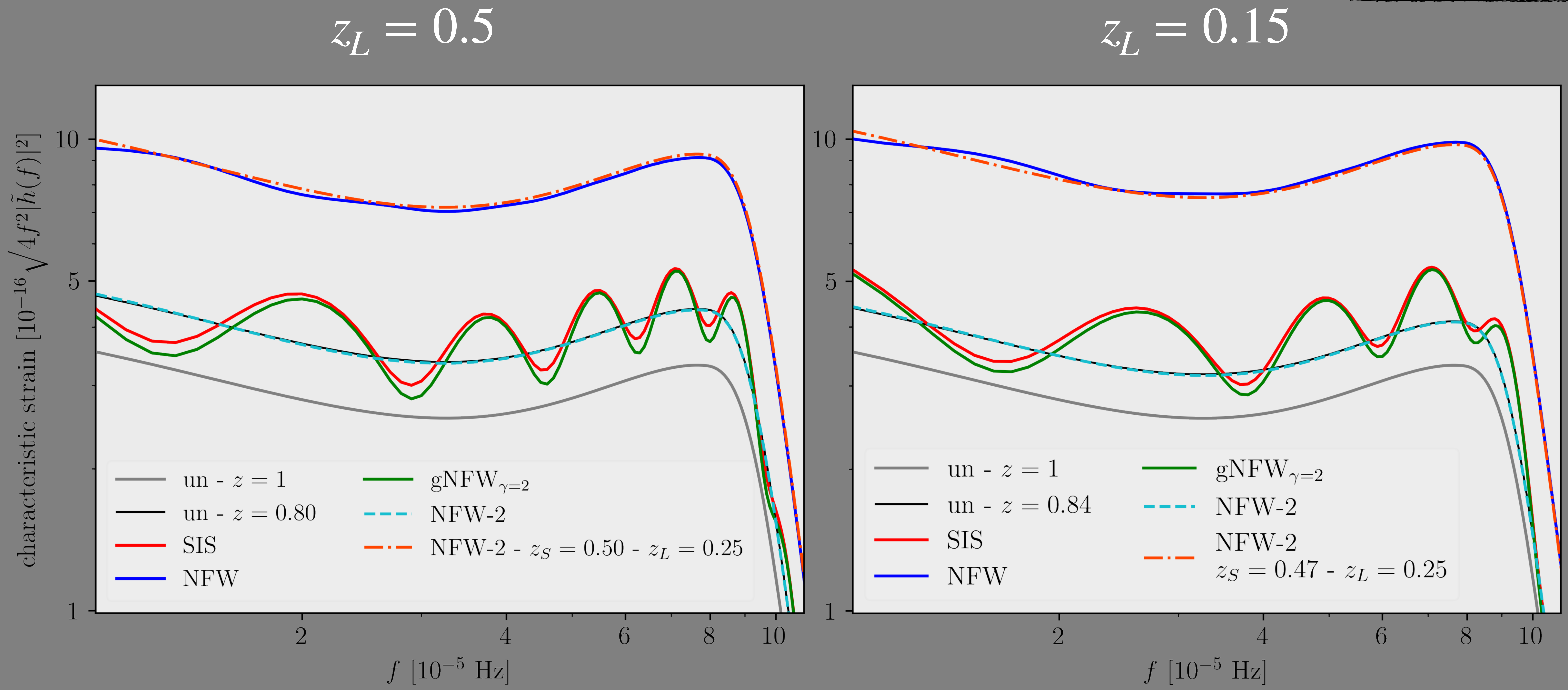
High precision lens modelling

Lensed waveforms

$z_S = 1$

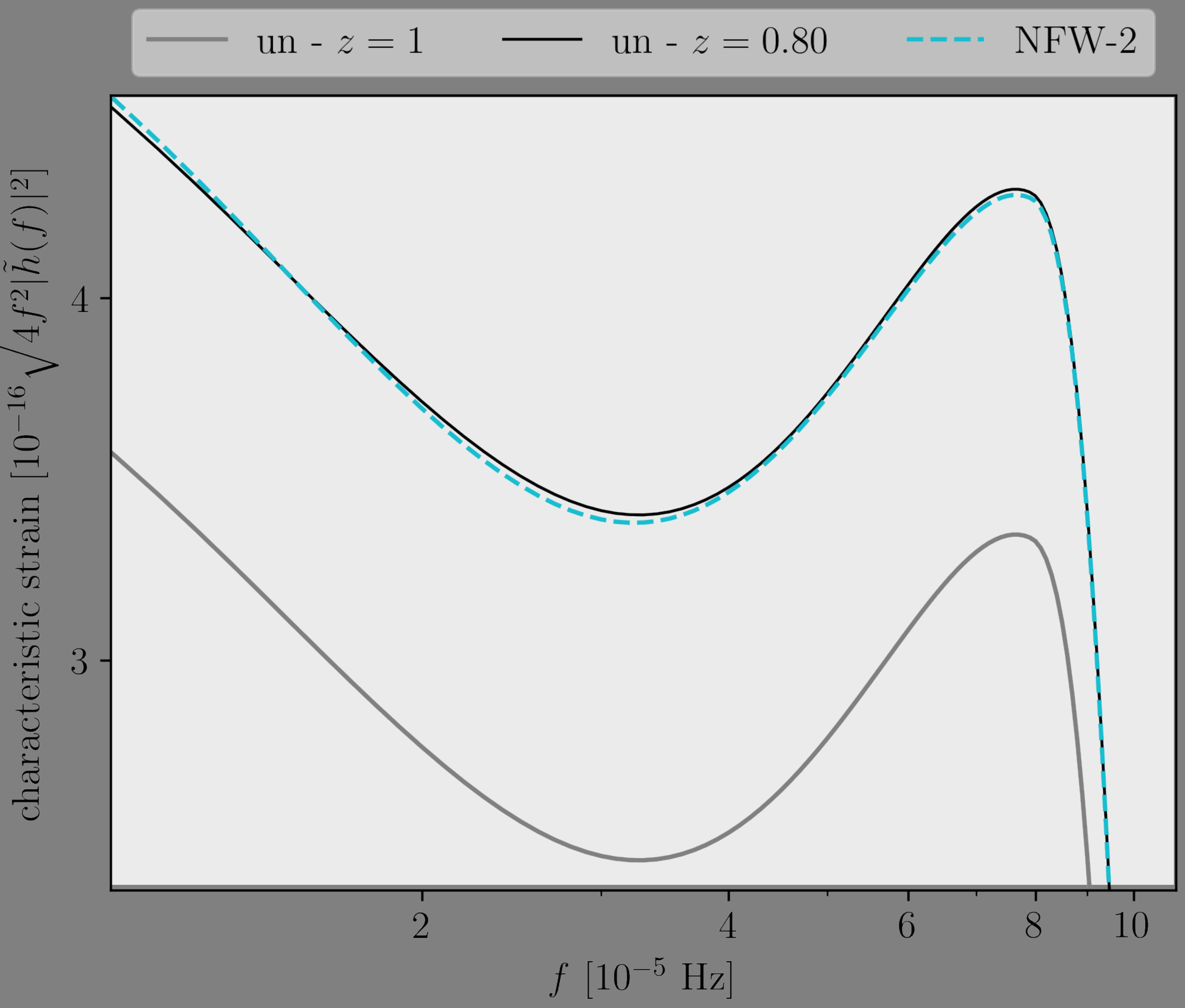
$M_s = 10^8\ M_\odot$

$M_L(r_c) = 10^9\ M_\odot$



Unlensed vs lensed

Lensed waveforms



$\rho \approx 220$ SNR of the signal

$\dfrac{\rho}{\rho_{opt}} = 1 - 4 \cdot 10^{-7}$ lensed / unlensed

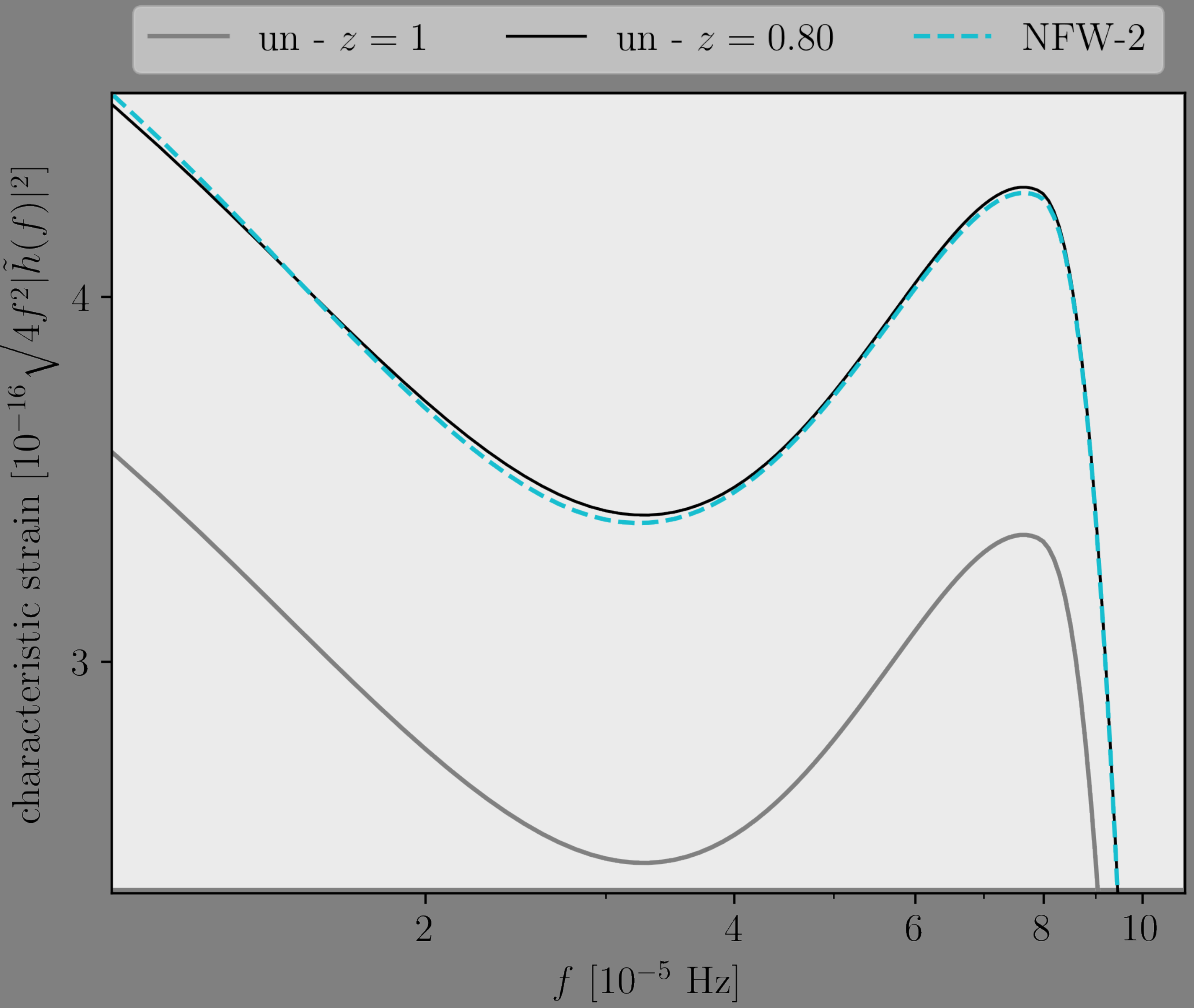
$\Delta\chi^2 \approx 14.2$ 3 free parameters

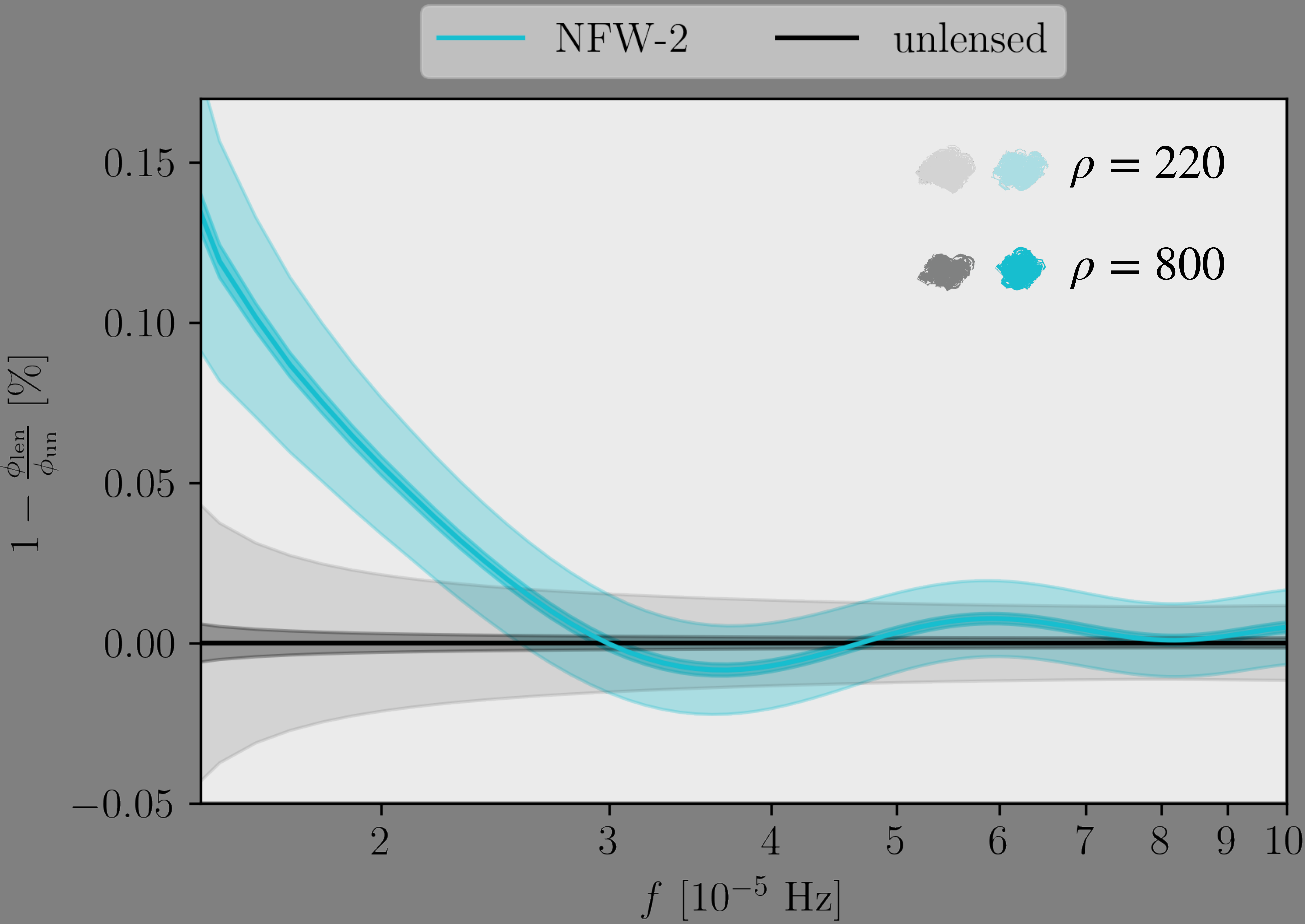
$\dfrac{\rho}{\rho_{opt}} = 1 - 1.5 \cdot 10^{-4}$ 3σ threshold

We would need $\rho \approx 4000$

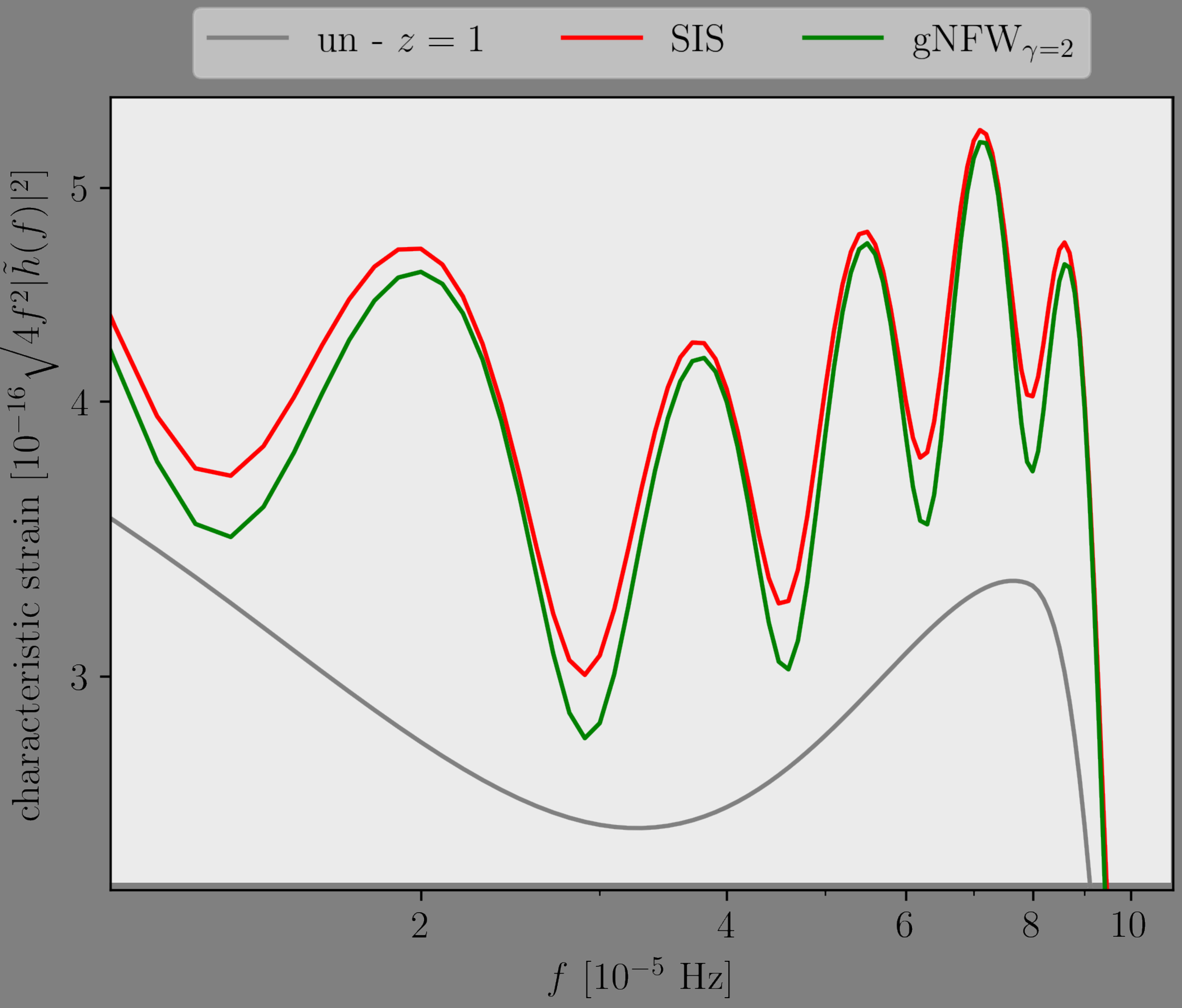
Unlensed vs lensed

Lensed waveforms





Constraining lens models



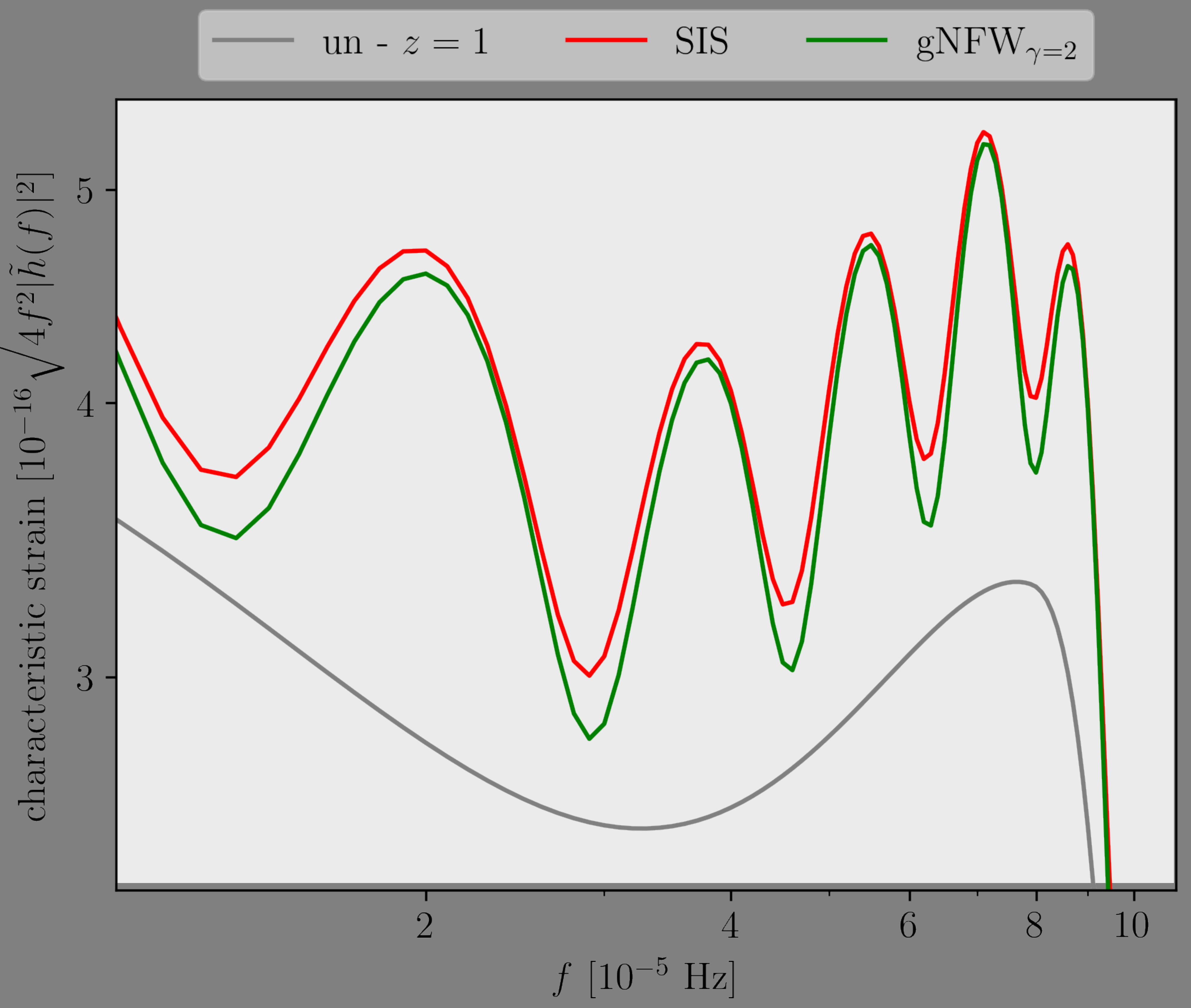
$\rho \approx 100$ SNR of the signal

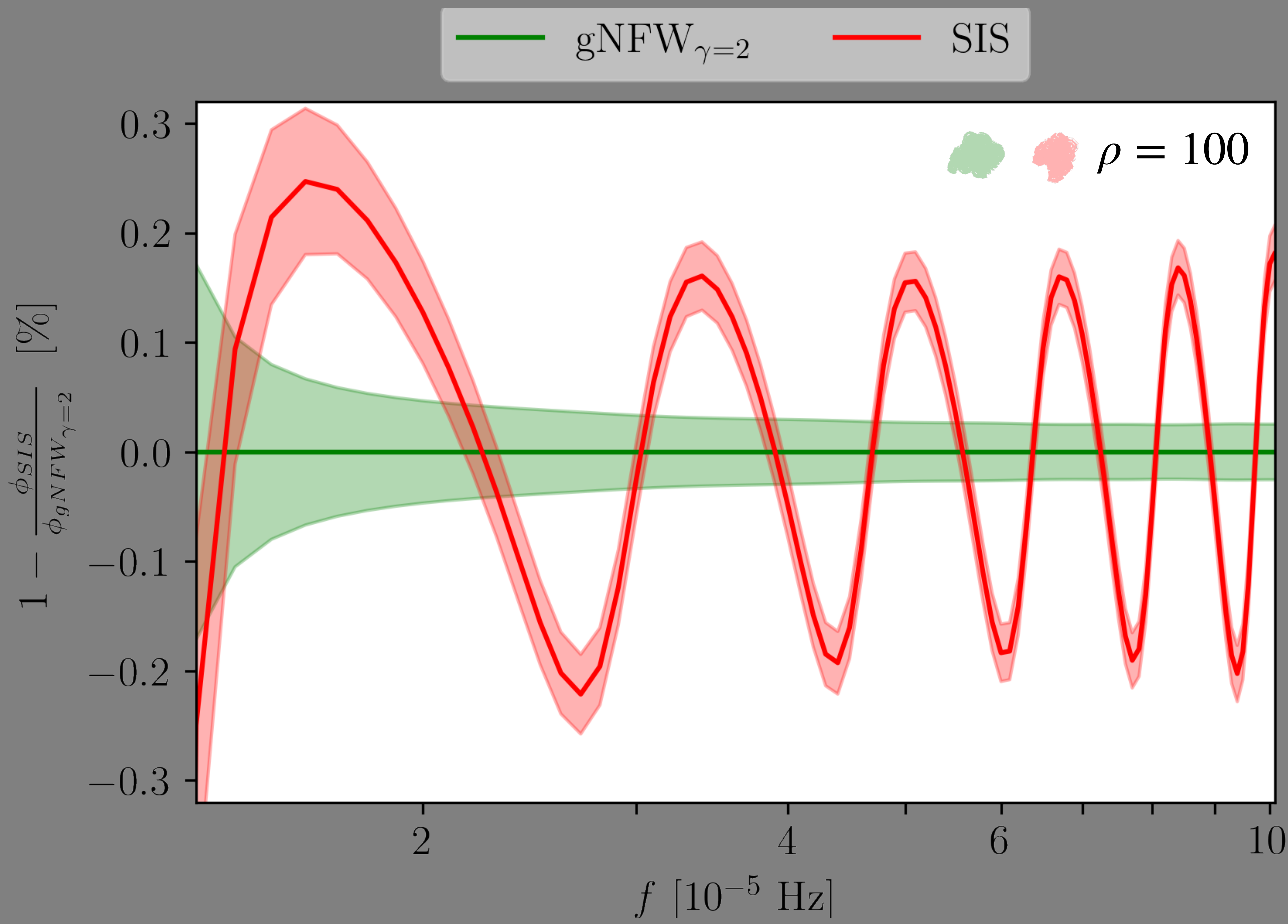
$\frac{\rho}{\rho_{opt}} = 0.9869$ SIS / gNFW$_{\gamma=2}$

$\Delta\chi^2 \approx 11.8$ 2 free parameters

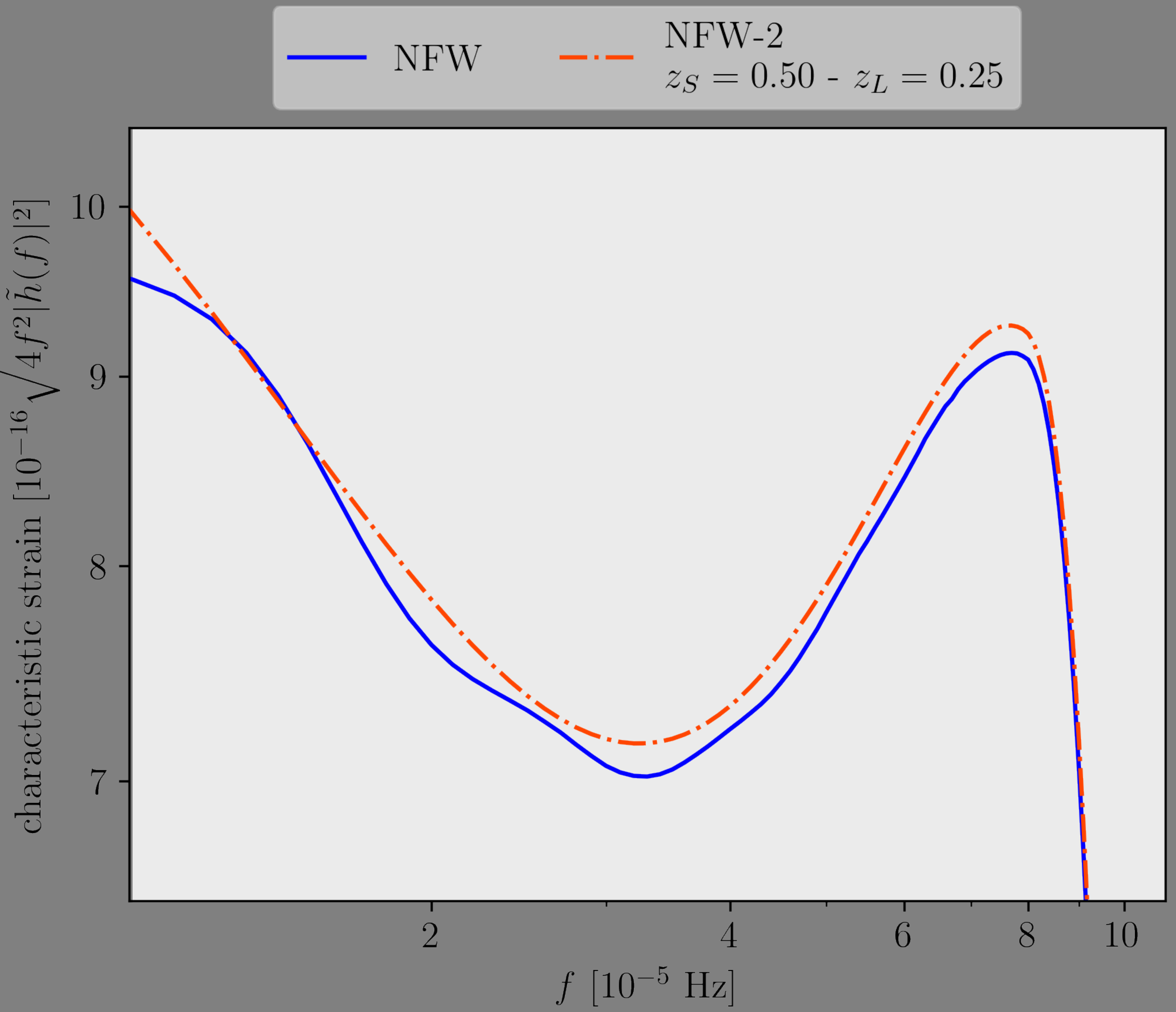
$\frac{\rho}{\rho_{opt}} = 0.9994$ 3σ threshold

Constraining lens models





Constraining lens models



$\rho \approx 220$ SNR of the signal

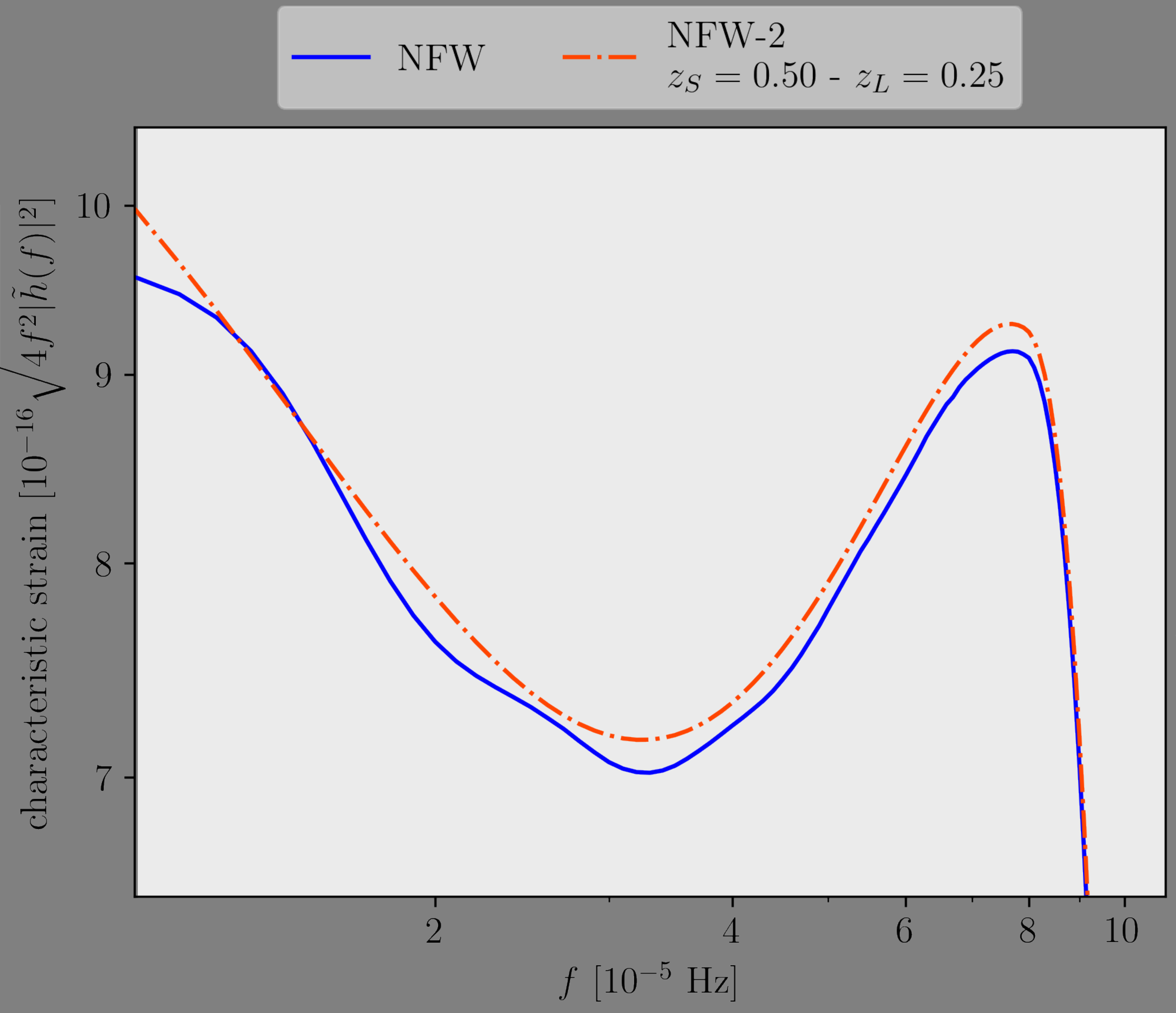
$\frac{\rho}{\rho_{opt}} = 1 - 1.4 \cdot 10^{-6}$ NFW / NFW-2

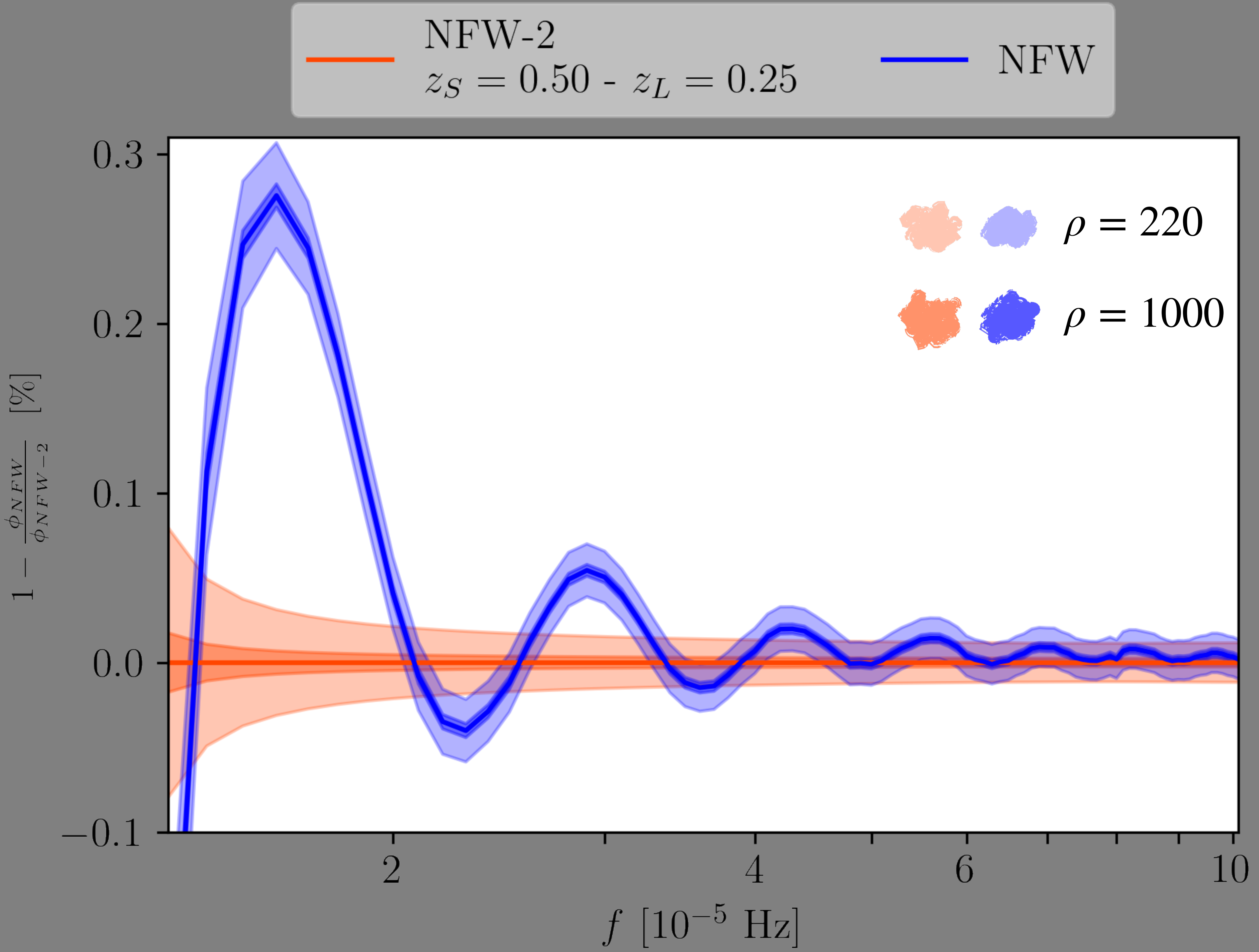
$\Delta\chi^2 \approx 14.2$ 3 free parameters

$\frac{\rho}{\rho_{opt}} = 1 - 1.4 \cdot 10^{-4}$ 3σ threshold

We would need $\rho \approx 2200$

Constraining lens models





Conclusions 3/3

1. Lensed events can be misinterpreted by unlensed one
2. Studying the phase of the signal is more effective than matched filtering
3. We can differentiate between lens models
4. Differentiating between models is useful to study dark matter/dark energy content