Mass-Sheet degeneracy in Gravitational Wave Lensing

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1. Mass-Sheet Degeneracy

The MSD is a well-known problem in lensing of light.
It consists on [1]

- Scaling of lens mass
  (κ - surface mass density)

$$\kappa \rightarrow \kappa_\lambda = \lambda\kappa + (1-\lambda)$$

- Scaling of angles

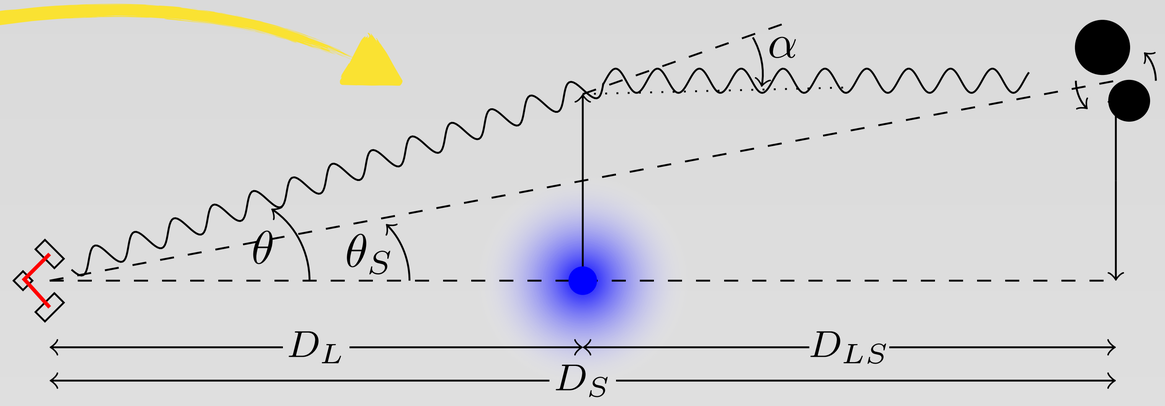
$$\vec{\alpha} \rightarrow \vec{\alpha}_\lambda = \lambda\vec{\alpha} + (1-\lambda)\vec{\theta}$$
$$\vec{\theta}_S \rightarrow \vec{\theta}_{S,\lambda} = \lambda\vec{\theta}_S$$

λ is the transformation/ degeneracy parameter
$\lambda = 1 \Rightarrow$ no transformation

Re-parametrisation of lens parameter:

$$\theta_S \rightarrow y = \frac{\theta_S}{\theta_E}$$

where
θ_E: Einstein radius



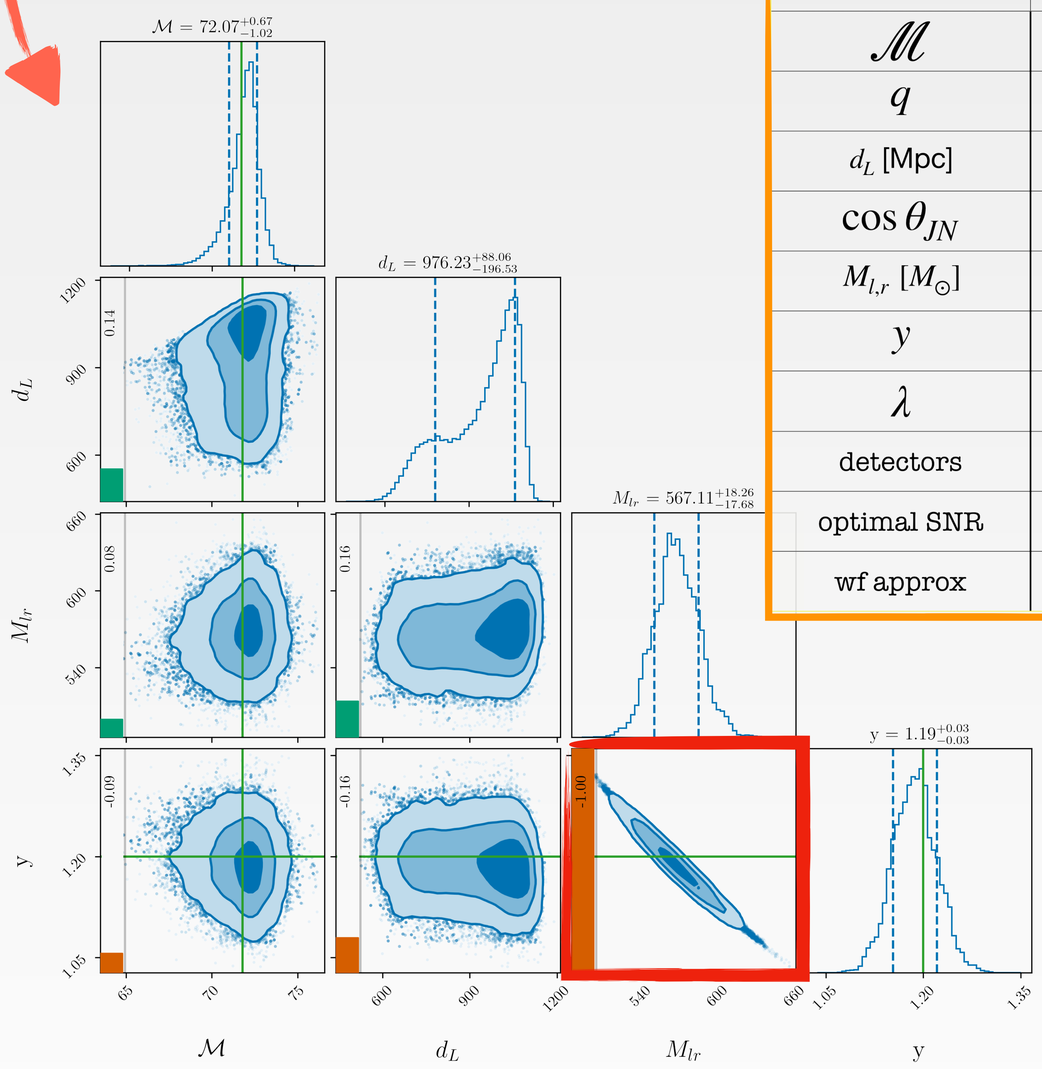
2. Problem

- Observables are preserved!
- Biased estimations of lens parameters
- Biased estimation of cosmological parameter

3. Method

Exploiting the work done in [2], we:

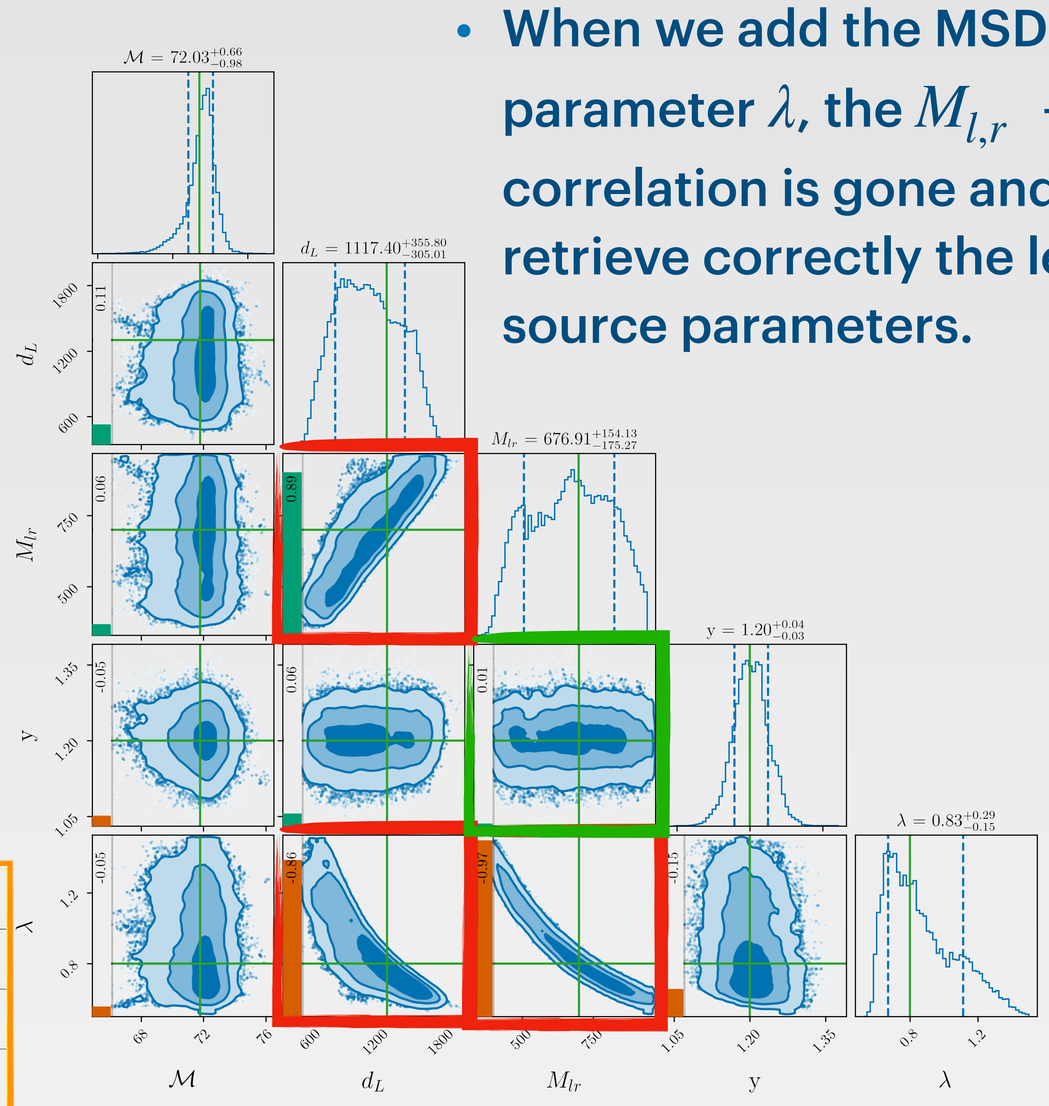
1. Rewrite the amplification factor [3] to include the MSD parameter: $\tilde{h}_L(f) = \tilde{h}(f) \cdot F(f, y, \lambda)$
2. Simulate a BBH lensed event with $\lambda \neq 1$
3. Study the event through 2 PE analysis:
   1. MSD parameter fixed to $\lambda = 1$
   2. MSD parameter free



Parameter	Value
$\mathscr{M}$	71.78
q	0.94
d_L [Mpc]	1300
$\cos\theta_{JN}$	0.95
$M_{l,r}$ [$M_\odot$]	700
y	1.2
λ	0.8
detectors	H1,L1,V1
optimal SNR	78
wf approx	IMRPhenomXP

4. Analysis

- In the "standard" PE analysis, the lens mass and the source distance are biased, as we can see from the first plot (bottom left).
- When we add the MSD parameter λ, the $M_{l,r} - y$ correlation is gone and we retrieve correctly the lens and source parameters.



5. Conclusions

Studying the MSD is important to:

1. set correct uncertainties of lens parameters
2. characterise lens model
3. astrophysical and cosmological studies

In this work we were able to properly recognise an external presence of mass and reconstruct the lens parameters correctly.

Bibliography

[1] E. E. Falco, M. V. Gorenstein, and I. I. Shapiro, ApJ289, L1 (1985)
[2] P. Cremonese, J.M. Ezquiaga,V. Salzano, Phys.Rev.D 104 (2021) 2, 023503
[3] T. T. Nakamura and S. Deguchi, "Progress of Theoretical Physics Supplement" 133, 137 (1999).

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